TAMA Modecleaner alignment error signals
Version 1.3

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1 Introduction

The effects of mirror misorientations[1] in interferometers with three or more mirrors cannot easily be determined analytically in the general case. Therefore a MATHEMATICA program for numerical 3-D ray tracing was written. It uses geometrical optics to find the axis of the eigenmode for a given combination of reflecting and/or refracting plane and spherical surfaces. A similar program is described and printed in Appendix E3 of MPQ243. The term 'beam' in this text refers to the geometrical axis of a beam, without taking into account the transverse shape or optical phase.

2 The modecleaner cavities

Figure 1 shows a schematic view of the TAMA modecleaner cavity seen from above, together with the coordinate system adopted in this section.

The cavity consists of two flat mirrors ($\mathrm{M_a}$ and $\mathrm{M_b}$) that are separated by a relatively short distance (20 cm), and a curved mirror $\mathrm{M_c}$ with radius of curvature $R = 15\,\mathrm{m}$, which is at a distance $L = 9.738\,\mathrm{m}$ from the (center between the) flat mirrors. The beam enters through $\mathrm{M_a}$ and travels clockwise to $\mathrm{M_b}$, $\mathrm{M_c}$, $\mathrm{M_a}$, etc.

$\mathrm{M_e}$ is the location of the photodetector behind the end mirror (in the following called 'end detector' for short), which is at a distance of 2.08 m behind mirror $\mathrm{M_c}$.

There are four beams of interest leaving the cavity. The main output beam is '$\mathrm{Out_b}$' which goes to the interferometer and the stabilization of which is the main purpose of the modecleaner. There are two beams coming from $\mathrm{M_a}$: the directly reflected input beam and the beam '$\mathrm{Out_a}$' which is a fraction of the cavity eigenmode. The cavity is well aligned to the incoming beam (which we consider fixed), if these two beams are perfectly superimposed. By taking two quadrant diodes with two different lens systems and appropriately demodulating their outputs, we can obtain four independent error signals, similar to the case of a simple two-mirror Fabry-Perot cavity. The longitudinal locking signal is also obtained from these two interfering beams (using the Pound-Drever-Hall scheme).

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[1] To avoid confusion, we call a mirror or other component **misoriented** in this section, if its angular orientation differs from its reference orientation. The resulting movement of beams (e.g. cavity eigenmodes) will be called **misalignment**. An interferometer is called **well-aligned** if there are neither misorientations nor misalignments, i.e. components as well as beams are in their reference positions.

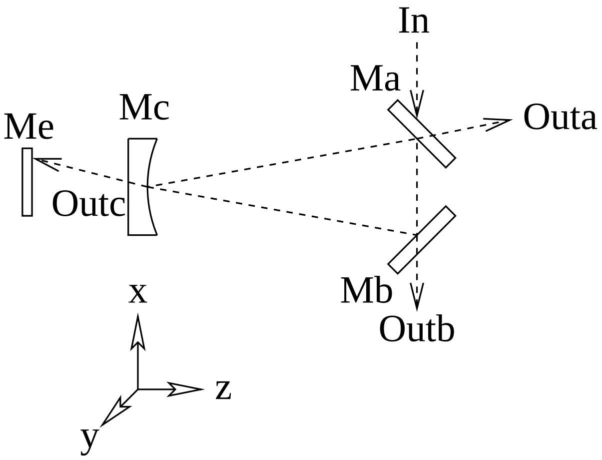

Figure 1: Schematic diagram of the TAMA modecleaner cavity seen from above.

In the three-mirror cavity, there are two additional degrees of freedom. They are linear combinations of movements of all three mirrors, as computed below. We will call them "neutral modes" because they have no influence on either the interference of the incoming beam with the cavity eigenmode nor on the outgoing beam. They can be used to control the spot position on the far mirror $\mathrm{M_c}$. This is described in more detail below.

In the ray-tracing program, we first compute the well-aligned case (i.e., all mirrors are hit in their center) as reference. We call $\mathrm{P_a}$, $\mathrm{P_b}$ and $\mathrm{P_c}$ the points where the axis of the eigenmode intersects the mirrors $\mathrm{M_a}$, $\mathrm{M_b}$ and $\mathrm{M_c}$, respectively. After misorienting one particular mirror by the small angle ε, we recompute the eigenmode axis, compare it with the well-aligned case and divide the difference by ε. The main results are:

- The shifts of the spots $\mathrm{P_a}$, $\mathrm{P_b}$ and $\mathrm{P_c}$.
- The angles γ_a, γ_b and γ_c between the beams '$\mathrm{Out_a}$', '$\mathrm{Out_b}$', '$\mathrm{Out_c}$' and their respective references. For the vertical misalignments which are considered separately, we call these angles δ_a, δ_b and δ_c, respectively. We also compute the angle γ_d (δ_d) between the directly reflected incoming beam and its reference for the case that $\mathrm{M_a}$ is misoriented.
- For '$\mathrm{Out_a}$' in the case of misorienting $\mathrm{M_a}$, we also compute the angle $\gamma'_\mathrm{a} = \gamma_\mathrm{a} - \gamma_\mathrm{d}$, which is the angle between the beam leaving the cavity and the directly reflected beam, because this is the angle between the interfering wavefronts that is detected by the quadrant diode.
- As described above we finally compute the angle θ which describes the 'character' of the misalignment at the waist. It is given by $\theta^\mathrm{w} = \arctan(\gamma'_\mathrm{a} z_\mathrm{R}/\Delta z_\mathrm{waist})$.

The waist of the cavity eigenmode is located halfway between the mirrors $\mathrm{M_a}$ and

$\mathrm{M_b}$. Its Rayleigh range is given by

$$z_\mathrm{R} = \sqrt{\frac{L_\mathrm{RT}}{2}\left(R - \frac{L_\mathrm{RT}}{2}\right)} = 7.126\,\mathrm{m}, \tag{1}$$

where $L_\mathrm{RT}/2$ is one half of the round-trip distance:

$$L_\mathrm{RT}/2 = \sqrt{L^2 + d^2/4} + d/2 = 9.8385\,\mathrm{m}. \tag{2}$$

3 Horizontal misalignments:

By ‘horizontal’ misalignments we mean that a mirror is rotated around the vertical axis, i.e. beam spots move horizontally (in the plane of the modecleaner cavity). Angles are counted as positive if a mirror is rotated *clockwise*, if seen from above (as in Figure 1).

We introduce the common- and differential mode motion of mirrors $\mathrm{M_a}$ and $\mathrm{M_b}$ by defining angles α_+ and α_-. Furthermore we introduce the “neutral” mode α_n as follows:

$$\begin{array}{c|c|c|c} & \mathrm{M_a} & \mathrm{M_b} & \mathrm{M_c} \\ \hline \alpha_+ & \alpha_\mathrm{a} = \alpha_+ & \alpha_\mathrm{b} = \alpha_+ & \alpha_\mathrm{c} = 0 \\ \alpha_- & \alpha_\mathrm{a} = \alpha_- & \alpha_\mathrm{b} = -\alpha_- & \alpha_\mathrm{c} = 0 \\ \alpha_\mathrm{n} & \alpha_\mathrm{a} = \alpha_\mathrm{n} & \alpha_\mathrm{b} = \alpha_\mathrm{n} & \alpha_\mathrm{c} = 0.701\alpha_\mathrm{n} \end{array} \tag{3}$$

$$\alpha_+ = \frac{\alpha_\mathrm{a} + \alpha_\mathrm{b}}{2}, \tag{4}$$

$$\alpha_- = \frac{\alpha_\mathrm{a} - \alpha_\mathrm{b}}{2}, \tag{5}$$

The results of the raytracing program are given in Table 1.

Cause	$\mathrm{P_a}$ Δx	$\mathrm{P_a}$ Δz	waist Δz	$\mathrm{Out_a}$ γ_a	$\mathrm{Out'_a}$ $\gamma'_\mathrm{a} = \gamma_\mathrm{a} - \gamma_\mathrm{d}$	θ^w
α_a	$-9.740\,\mathrm{m} \cdot \alpha_\mathrm{a}$	$9.840\,\mathrm{m} \cdot \alpha_\mathrm{a}$	$9.739\,\mathrm{m} \cdot \alpha_\mathrm{a}$	$0.981\,\alpha_\mathrm{a}$	$-1.019\,\alpha_\mathrm{a}$	$-36.72°$
α_b	$9.538\,\mathrm{m} \cdot \alpha_\mathrm{b}$	$-9.636\,\mathrm{m} \cdot \alpha_\mathrm{b}$	$-9.739\,\mathrm{m} \cdot \alpha_\mathrm{b}$	$-1.019\,\alpha_\mathrm{b}$	$-1.019\,\alpha_\mathrm{b}$	$36.72°$
α_c	$0.288\,\mathrm{m} \cdot \alpha_\mathrm{c}$	$-0.291\,\mathrm{m} \cdot \alpha_\mathrm{c}$	$0.000\,\mathrm{m} \cdot \alpha_\mathrm{c}$	$2.906\,\alpha_\mathrm{c}$	$2.906\,\alpha_\mathrm{c}$	$-90.00°$
α_+	$-0.202\,\mathrm{m} \cdot \alpha_+$	$0.204\,\mathrm{m} \cdot \alpha_+$	$0.000\,\mathrm{m} \cdot \alpha_+$	$-0.039\,\alpha_+$	$-2.039\,\alpha_+$	$90.00°$
α_-	$-19.278\,\mathrm{m} \cdot \alpha_-$	$19.477\,\mathrm{m} \cdot \alpha_-$	$19.477\,\mathrm{m} \cdot \alpha_-$	$2.000\,\alpha_-$	$0.000\,\alpha_-$	$0.00°$
α_n	$0.000\,\mathrm{m} \cdot \alpha_\mathrm{n}$	$0.000\,\mathrm{m} \cdot \alpha_\mathrm{n}$	$0.000\,\mathrm{m} \cdot \alpha_\mathrm{n}$	$2.000\,\alpha_\mathrm{n}$	$0.000\,\alpha_\mathrm{n}$	—

Cause	$\mathrm{P_b}$ Δx	$\mathrm{P_b}$ Δz	$\mathrm{Out_b}$ γ_b	$\mathrm{P_c}$ Δx	$\mathrm{Out_c}$ γ_c	d. refl. γ_d	End Δx
α_a	$9.538\,\mathrm{m} \cdot \alpha_\mathrm{a}$	$9.637\,\mathrm{m} \cdot \alpha_\mathrm{a}$	$1.019\,\alpha_\mathrm{a}$	$-0.291\,\mathrm{m} \cdot \alpha_\mathrm{a}$	$-1.019\,\alpha_\mathrm{a}$	$2 \cdot \alpha_\mathrm{a}$	$-2.411\,\mathrm{m} \cdot \alpha_\mathrm{a}$
α_b	$-9.740\,\mathrm{m} \cdot \alpha_\mathrm{b}$	$-9.841\,\mathrm{m} \cdot \alpha_\mathrm{b}$	$1.019\,\alpha_\mathrm{b}$	$-0.291\,\mathrm{m} \cdot \alpha_\mathrm{b}$	$0.981\,\alpha_\mathrm{b}$	$0 \cdot \alpha_\mathrm{b}$	$1.749\,\mathrm{m} \cdot \alpha_\mathrm{b}$
α_c	$0.288\,\mathrm{m} \cdot \alpha_\mathrm{c}$	$0.291\,\mathrm{m} \cdot \alpha_\mathrm{c}$	$-2.906\,\alpha_\mathrm{c}$	$28.596\,\mathrm{m} \cdot \alpha_\mathrm{c}$	$2.906\,\alpha_\mathrm{c}$	$0 \cdot \alpha_\mathrm{c}$	$34.642\,\mathrm{m} \cdot \alpha_\mathrm{c}$
α_+	$-0.202\,\mathrm{m} \cdot \alpha_+$	$-0.204\,\mathrm{m} \cdot \alpha_+$	$2.039\,\alpha_+$	$-0.581\,\mathrm{m} \cdot \alpha_+$	$-0.039\,\alpha_+$	$2 \cdot \alpha_+$	$-0.662\,\mathrm{m} \cdot \alpha_+$
α_-	$19.278\,\mathrm{m} \cdot \alpha_-$	$19.477\,\mathrm{m} \cdot \alpha_-$	$0.000\,\alpha_-$	$0.000\,\mathrm{m} \cdot \alpha_-$	$-2.000\,\alpha_-$	$2 \cdot \alpha_-$	$-4.160\,\mathrm{m} \cdot \alpha_-$
α_n	$0.000\,\mathrm{m} \cdot \alpha_\mathrm{n}$	$0.000\,\mathrm{m} \cdot \alpha_\mathrm{n}$	$0.000\,\alpha_\mathrm{n}$	$19.478\,\mathrm{m} \cdot \alpha_\mathrm{n}$	$2.000\,\alpha_\mathrm{n}$	$2 \cdot \alpha_\mathrm{n}$	$23.639\,\mathrm{m} \cdot \alpha_\mathrm{n}$

Table 1: Results of the ray-tracing program for horizontal misalignments of the TAMA modecleaner.

4 Vertical misalignments:

In the modecleaners, the horizontal and vertical axes are *not* similar. The results of the raytracing program for vertical misalignments are given in Table 2. Note, for example, that a small vertical tilt β_a of the input mirror $\mathrm{M_a}$ (which is hit under approximately 45° from the incoming beam) causes a deflection of the reflected beam by only $\delta_\mathrm{d} = 1.43\beta_\mathrm{a}$ as compared to $\gamma_\mathrm{d} = 2\alpha_\mathrm{a}$ in the horizontal case. Another example is the tilt of $\mathrm{M_c}$ which, if horizontal, causes a pure angular misalignment at the waist. A vertical tilt of $\mathrm{M_c}$, on the other hand, shifts the cavity eigenmode downwards parallely, without changing any angles. Angles are now counted as positive when the mirror normal moves downward from the reference direction.

As before we introduce the common- and differential mode motion of mirrors $\mathrm{M_a}$ and $\mathrm{M_b}$ by defining angles β_+ and β_-. Furthermore we introduce the "neutral" mode β_n as follows:

$$\begin{array}{c|c|c|c} & \mathrm{M_a} & \mathrm{M_b} & \mathrm{M_c} \\ \hline \beta_+ & \beta_\mathrm{a} = \beta_+ & \beta_\mathrm{b} = \beta_+ & \beta_\mathrm{c} = 0 \\ \beta_- & \beta_\mathrm{a} = \beta_- & \beta_\mathrm{b} = -\beta_- & \beta_\mathrm{c} = 0 \\ \beta_\mathrm{n} & \beta_\mathrm{a} = \beta_\mathrm{n} & \beta_\mathrm{b} = \beta_\mathrm{n} & \beta_\mathrm{c} = -0.4986\beta_\mathrm{n} \end{array} \tag{6}$$

$$\beta_+ = \frac{\beta_\mathrm{a} + \beta_\mathrm{b}}{2}, \tag{7}$$

$$\beta_- = \frac{\beta_\mathrm{a} - \beta_\mathrm{b}}{2}, \tag{8}$$

The results of the raytracing program are given in Table 2.

5 Degrees of freedom

The most important alignment task is to superimpose the axis of the cavity eigenmode with the axis of the incoming beam. This requires the control of four degrees of freedom. For this purpose, in the differential wavefront sensing method, we place two quadrant detectors with different lens systems in the beam reflected from $\mathrm{M_a}$. The interference between the directly reflected incoming beam, which is phase modulated at an RF frequency, and the beam '$\mathrm{Out_a}$' leaking out of the cavity contains enough information to lock the cavity longitudinally and to obtain alignment error signals for those four degrees of freedom that determine the superposition of the incoming beam and the cavity eigenmode.

In particular, we now assume all mirrors to be slightly misoriented and compute the combined signals which are obtained by demodulating the outputs of two quadrant detectors, one (called X_I) with $\Phi = 0°$ and the other one (called X_Q) with $\Phi = 90°$ of extra phase shift. We scale parallel shifts Δy or Δz with the appropriate factor z_R and obtain for horizontal misalignments:

$$X_I = -1.019\,\alpha_\mathrm{a} - 1.019\,\alpha_\mathrm{b} + 2.906\,\alpha_\mathrm{c} = -1.019\,\alpha_+ + 2.906\,\alpha_\mathrm{c}, \tag{9}$$

$$X_Q = 1.366\,\alpha_\mathrm{a} - 1.366\,\alpha_\mathrm{b} = 1.366\,\alpha_-, \tag{10}$$

Cause	$\mathrm{P_a}$	waist	$\mathrm{Out_a}$	$\mathrm{Out'_a}$	θ^{w}
	Δy	Δy	δ_{a}	$\delta'_{\mathrm{a}} = \delta_{\mathrm{a}} - \delta_{\mathrm{d}}$	
β_{a}	$-3.670\,\mathrm{m}\cdot\beta_{\mathrm{a}}$	$-3.740\,\mathrm{m}\cdot\beta_{\mathrm{a}}$	$-0.718\,\beta_{\mathrm{a}}$	$0.704\,\beta_{\mathrm{a}}$	$-53.27°$
β_{b}	$-3.810\,\mathrm{m}\cdot\beta_{\mathrm{b}}$	$-3.740\,\mathrm{m}\cdot\beta_{\mathrm{b}}$	$-0.704\,\beta_{\mathrm{b}}$	$-0.704\,\beta_{\mathrm{b}}$	$53.27°$
β_{c}	$-15.000\,\mathrm{m}\cdot\beta_{\mathrm{c}}$	$-15.000\,\mathrm{m}\cdot\beta_{\mathrm{c}}$	$0.000\,\beta_{\mathrm{c}}$	$0.000\,\beta_{\mathrm{c}}$	$0.0°$
β_{+}	$-7.480\,\mathrm{m}\cdot\beta_{+}$	$-7.480\,\mathrm{m}\cdot\beta_{+}$	$-1.421\,\beta_{+}$	$0.000\,\beta_{+}$	$0.0°$
β_{-}	$0.141\,\mathrm{m}\cdot\beta_{-}$	$0.000\,\mathrm{m}\cdot\beta_{-}$	$-0.014\,\beta_{-}$	$1.407\,\beta_{-}$	$90.0°$
β_{n}	$0.000\,\mathrm{m}\cdot\beta_{\mathrm{n}}$	$0.000\,\mathrm{m}\cdot\beta_{\mathrm{n}}$	$-1.421\,\beta_{\mathrm{n}}$	$0.000\,\beta_{\mathrm{n}}$	—

Cause	$\mathrm{P_b}$	$\mathrm{Out_b}$	$\mathrm{P_c}$	$\mathrm{Out_c}$	d. refl.	End
	Δy	δ_{b}	Δy	δ_{c}	δ_{d}	Δy
β_{a}	$-3.810\,\mathrm{m}\cdot\beta_{\mathrm{a}}$	$0.704\,\beta_{\mathrm{a}}$	$-10.661\,\mathrm{m}\cdot\beta_{\mathrm{a}}$	$0.704\,\beta_{\mathrm{a}}$	$1.42\cdot\beta_{\mathrm{a}}$	$-12.125\,\mathrm{m}\cdot\beta_{\mathrm{a}}$
β_{b}	$-3.670\,\mathrm{m}\cdot\beta_{\mathrm{b}}$	$-0.704\,\beta_{\mathrm{b}}$	$-10.661\,\mathrm{m}\cdot\beta_{\mathrm{b}}$	$0.718\,\beta_{\mathrm{b}}$	$0.00\cdot\beta_{\mathrm{b}}$	$-12.155\,\mathrm{m}\cdot\beta_{\mathrm{b}}$
β_{c}	$-15.000\,\mathrm{m}\cdot\beta_{\mathrm{c}}$	$0.000\,\beta_{\mathrm{c}}$	$-15.000\,\mathrm{m}\cdot\beta_{\mathrm{c}}$	$0.000\,\beta_{\mathrm{c}}$	$0\cdot\beta_{\mathrm{c}}$	$-15.000\,\mathrm{m}\cdot\beta_{\mathrm{c}}$
β_{+}	$-7.480\,\mathrm{m}\cdot\beta_{+}$	$0.000\,\beta_{+}$	$-21.323\,\mathrm{m}\cdot\beta_{+}$	$1.421\,\beta_{+}$	$-1.421\cdot\beta_{+}$	$-24.280\,\mathrm{m}\cdot\beta_{+}$
β_{-}	$-0.141\,\mathrm{m}\cdot\beta_{-}$	$1.407\,\beta_{-}$	$0.000\,\mathrm{m}\cdot\beta_{-}$	$-0.014\,\beta_{-}$	$-1.421\cdot\beta_{-}$	$0.030\,\mathrm{m}\cdot\beta_{-}$
β_{n}	$0.000\,\mathrm{m}\cdot\beta_{\mathrm{n}}$	$0.000\,\beta_{\mathrm{n}}$	$-13.843\,\mathrm{m}\cdot\beta_{\mathrm{n}}$	$1.421\,\beta_{\mathrm{n}}$	$-1.420\cdot\beta_{\mathrm{n}}$	$-16.800\,\mathrm{m}\cdot\beta_{\mathrm{n}}$

Table 2: Results of the ray-tracing program for vertical misalignments of the TAMA modecleaner.

and for vertical misalignments:

$$Y_I = 0.7035\,\beta_{\mathrm{a}} - 0.7035\,\beta_{\mathrm{b}} = 0.7035\,\beta_{-}, \tag{11}$$

$$Y_Q = -0.524\,\beta_{\mathrm{a}} - 0.524\,\beta_{\mathrm{b}} - 2.105\,\beta_{\mathrm{c}} = -0.524\,\beta_{+} - 2.105\,\beta_{\mathrm{c}}. \tag{12}$$

Using simple linear algebra we also find the "neutral" modes from these results. They are given in tables (3) and (6) above.

By inspecting tables 1 and 2 one finds that the main effect of these "neutral modes" is to shift the spot position on the far end mirror $\mathrm{M_c}$. Hence their control is not absolutely essential for the basic function of the modecleaner. In the present situation, there is neither feedback to $\mathrm{M_c}$ nor is the spot position on $\mathrm{M_c}$ monitored. One problem with this approach might be that according to equations (9) to (12), a small motion of $\mathrm{M_c}$ translates into error signals at the reflected light port which is 3…4 times larger than the error signal for a motion of the front mirrors of comparable amplitued. This huge error signal is then fed back the the *front* mirrors which are forced to compensate $\mathrm{M_c}$'s misorientation although they are not very efficient actuators for that purpose. Furthermore, such feedback shifts the spot position on $\mathrm{M_c}$ which due to the curved mirror surface again causes the whole circle to start. It is possible that some of the present dynamic range problems may be relieved by sensing and feeding back all degrees of freedom.

Because the main effect of the "neutral modes" is to shift the spot position on the far end mirror $\mathrm{M_c}$, the most straightforward approach is to sense this spot position (with a DC quadrant detector looking at the transmitted light). In tables 1 and 2 above, this shift is computed as Δx or Δy of point Pc, respectively.

However, the detector is located at a distance $e = 2.08\,\mathrm{m}$ behind $\mathrm{M_c}$, and the spot position detected is hence a linear combination[2] of Δx or Δy of point Pc, respectively with the angle δ_c.

This shift of the beam spot on the photodetector is also calculated by the program and printed as "End Δy" in Tables 1 and 2.

If we also scale it by the factor z_R for consistency (although the way of detection is different and hence an arbitrary scale factor appears anyway), and call the error signals thus obtained X_E and Y_E, we get

$$\begin{aligned}
X_E &= -0.3383\,\alpha_\mathrm{a} + 0.2455\,\alpha_\mathrm{b} + 4.8613\,\alpha_\mathrm{c} &(13)\\
&= -0.0464\,\alpha_+ - 0.2919\,\alpha_- + 4.8613\,\alpha_\mathrm{c}, &(14)\\
Y_E &= -1.701\,\beta_\mathrm{a} - 1.705\,\beta_\mathrm{b} - 2.105\,\beta_\mathrm{c} &(15)\\
&= -1.703\,\beta_+ + 0.002\,\beta_- - 2.105\,\beta_\mathrm{c}. &(16)
\end{aligned}$$

We see that for the horizontal direction, the additional error signal is dominated by the misorientation of $\mathrm{M_c}$ and could directly be fed back to that mirror, while for the vertical direction the signals are more mixed. However, they are still sufficiently linearly independent such that by inversion of the well-conditioned matrix error signals for all three mirrors can be found. This matrix inversion yields

$$\begin{array}{rcrrr}
\alpha_\mathrm{a} & = & X_E & -1.6726\,X_I & +1.4273\,X_Q \\
\alpha_\mathrm{b} & = & X_E & -1.6726\,X_I & -X_Q \\
\alpha_\mathrm{c} & = & 0.7015\,X_E & -0.0319\,X_I & +0.1498\,X_Q
\end{array} \qquad (17)$$

$$\begin{array}{rcrrr}
\beta_\mathrm{a} & = & -Y_E & +1.6785\,Y_I & +Y_Q \\
\beta_\mathrm{b} & = & -Y_E & -1.6785\,Y_I & +Y_Q \\
\beta_\mathrm{c} & = & 0.4987\,Y_E & -0.0015\,Y_I & -1.6186\,Y_Q
\end{array} \qquad (18)$$

where scaling factors of 1/0.301462 and 1/0.42418, respectively, have been applied to make the most common coefficients unity. Note that the present alignment system uses only a 2×2 matrix without X_E, Y_E, α_c and β_c.

6 Actuator transfer functions

The transfer functions from all actuators to all sensors have been measured by K. Arai. The results were fitted with LISO and are shown on the following pages.

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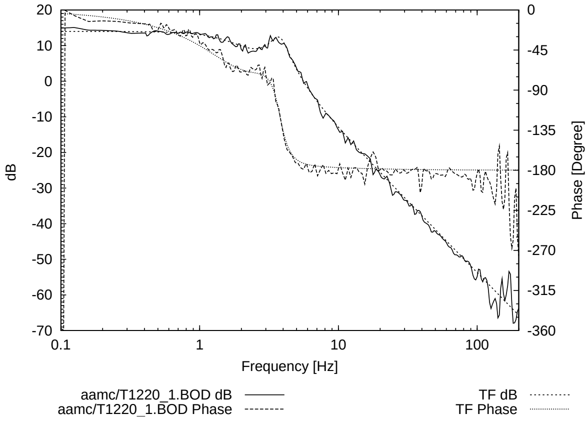
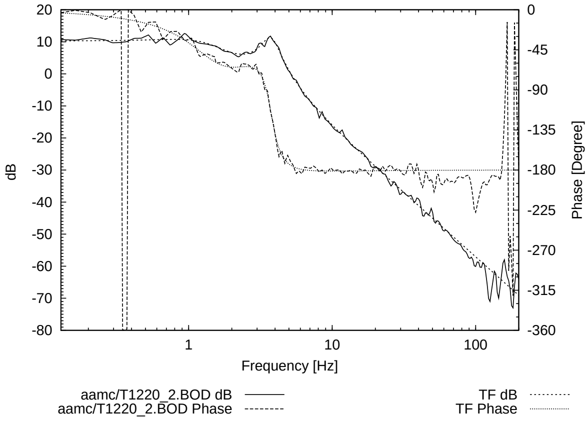
[2] Many thanks to K. Arai for noting this point.

6.1 Input Mirror Yaw coil ("m1xcoil")



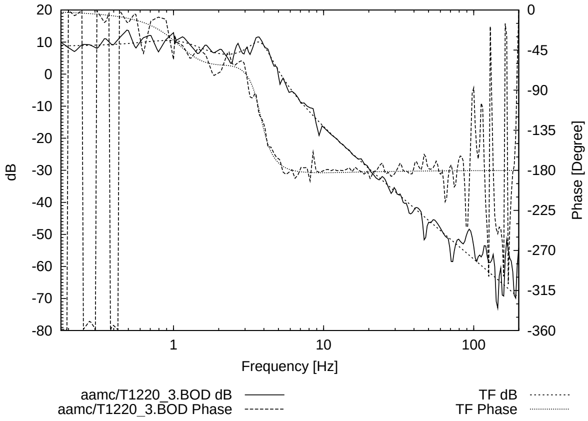
```
Pi Yaw error      T1220_1.BOD

pole0:f = 1.6798911 +- 75.2m (4.48%)
pole0:q = 636.47668m +- 21.6m (3.39%)
zero0:f = 3.1323094 +- 124.6m (3.98%)
zero0:q = 969.43027m +- 64m (6.6%)
pole1:f = 3.7858238 +- 25.9m (0.684%)
pole1:q = 3.3333594 +- 177.3m (5.32%)
factor = 5.0043609 +- 40.31m (0.805%)
```



```
Pi/2 Yaw error       T1220_2.BOD

pole0:f = 1.3598055 +- 43.77m (3.22%)
pole0:q = 830.24256m +- 19.97m (2.41%)
zero0:f = 2.4762582 +- 72.09m (2.91%)
zero0:q = 842.25894m +- 47.99m (5.7%)
pole1:f = 3.7674492 +- 19.53m (0.518%)
pole1:q = 3.711112 +- 128m (3.45%)
factor = 3.2644629 +- 29.84m (0.914%)
```



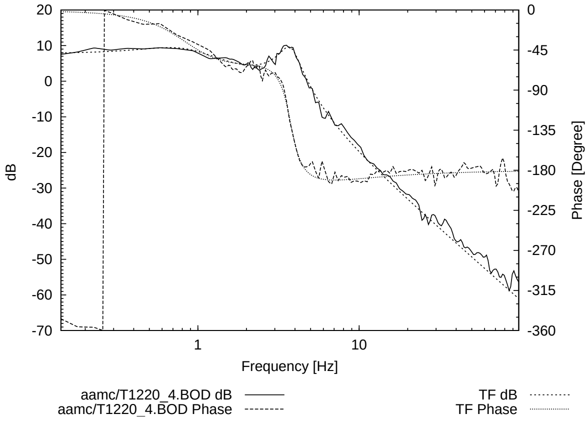
```
End QPD DC Yaw       T1220_3.BOD

pole0:f = 1.1584155 +- 66.98m (5.78%)
pole0:q = 948.86053m +- 53.28m (5.61%)
zero0:f = 2.0163492 +- 120.4m (5.97%)
zero0:q = 642.40736m +- 78.67m (12.2%)
pole1:f = 3.8080309 +- 46.85m (1.23%)
pole1:q = 3.0342956 +- 189.4m (6.24%)
factor = 2.7204019 +- 63.32m (2.33%)
```

Weighted Averages:

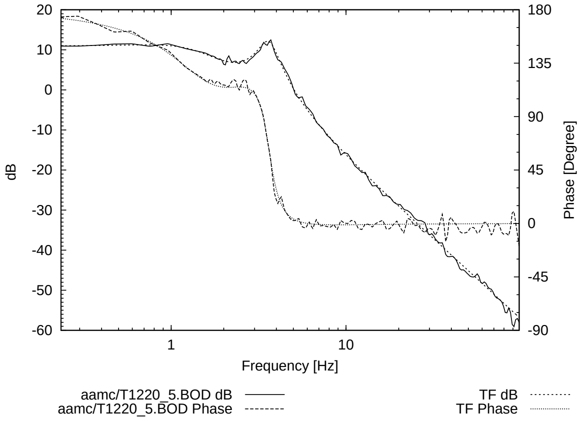
```
pole0:f = 1.372388
pole0:q = 0.755353
pole1:f = 3.777486
pole1:q = 3.455169
zero0:f = 2.508392
zero0:q = 0.840509
```

6.2 Output Mirror Yaw coil ("m2xcoil")



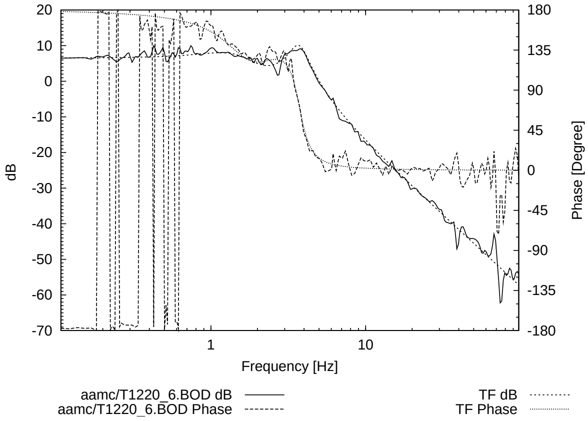
```
Pi Yaw error      T1220_4.BOD

pole0:f = 864.55282m +- 31.63m (3.66%)
pole0:q = 801.30084m +- 22.58m (2.82%)
zero0:f = 1.7498418 +- 67.69m (3.87%)
zero0:q = 436.75937m +- 27.63m (6.33%)
pole1:f = 3.7556255 +- 19.05m (0.507%)
pole1:q = 3.6996359 +- 118m (3.19%)
factor = 2.4554303 +- 33.68m (1.37%)
```



```
Pi/2 Yaw error       T1220_5.BOD

pole0:f = 1.3974404 +- 26.7m (1.91%)
pole0:q = 792.95797m +- 11.9m (1.5%)
zero0:f = 2.5625299 +- 44.11m (1.72%)
zero0:q = 877.60669m +- 27.81m (3.17%)
pole1:f = 3.630812 +- 9.79m (0.27%)
pole1:q = 3.9492515 +- 79.85m (2.02%)
factor = -3.5256953 +- -21.65m (0.614%)
```



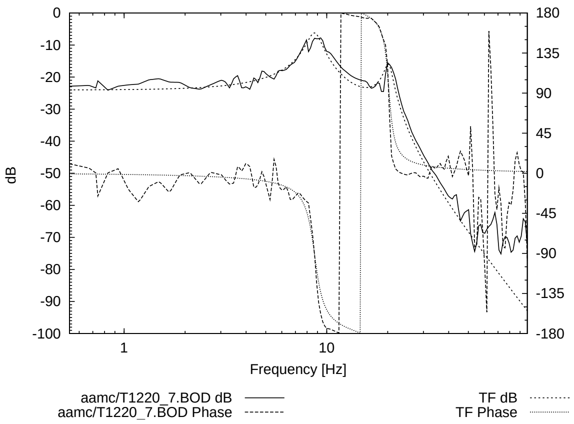
```
End QPD DC Yaw       T1220_6.BOD

pole0:f = 1.5309455 +- 68.42m (4.47%)
pole0:q = 1.0838276 +- 56.89m (5.25%)
zero0:f = 2.2872224 +- 93.83m (4.1%)
zero0:q = 1.0726809 +- 108.7m (10.1%)
pole1:f = 3.7250662 +- 31.92m (0.857%)
pole1:q = 3.6033149 +- 192.1m (5.33%)
factor = -2.1236194 +- -32.95m (1.55%)
```

Weighted Averages:

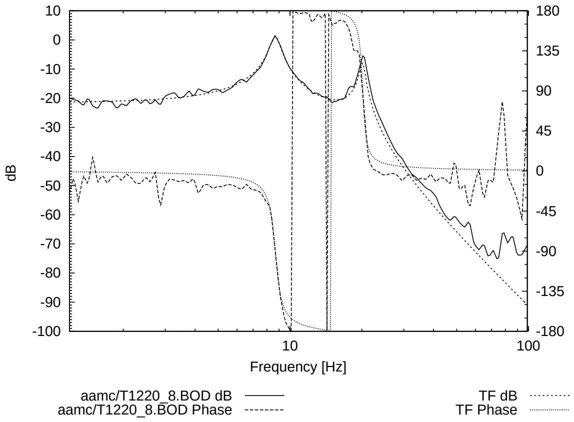
```
pole0:f = 1.204886
pole0:q = 0.804331
pole1:f = 3.661688
pole1:q = 3.842627
zero0:f = 2.316205
zero0:q = 0.669030
```

6.3 Input Mirror Pitch PZT ("m1ypzt")



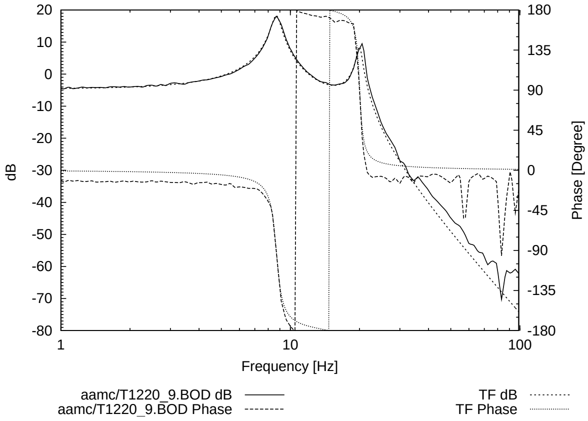
```
Pi Pitch error       T1220_7.BOD

pole0:f = 8.721352 +- 31.18m (0.357%)
pole0:q = 6.3320232 +- 330.2m (5.22%)
pole1:f = 20.249388 +- 100.9m (0.498%)
pole1:q = 11.065166 +- 1.296 (11.7%)
factor = 62.862325m +- 1.561m (2.48%)
```



```
Pi/2 Pitch error     T1220_8.BOD

pole0:f = 8.6836151 +- 12.55m (0.145%)
pole0:q = 11.122052 +- 405.8m (3.65%)
pole1:f = 20.153226 +- 42.54m (0.211%)
pole1:q = 20.638458 +- 1.887 (9.15%)
factor = 84.273794m +- 1.359m (1.61%)
```



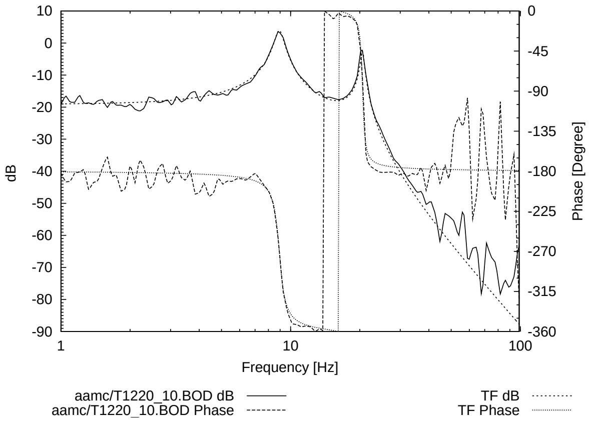
```
End QPD DC Pitch     T1220_9.BOD

pole0:f = 8.6874272 +- 10.29m (0.118%)
pole0:q = 11.243448 +- 337.4m (3%)
pole1:f = 20.031176 +- 32.46m (0.162%)
pole1:q = 19.667483 +- 1.306 (6.64%)
factor = 590.1168m +- 7.742m (1.31%)
```

Weighted Averages:

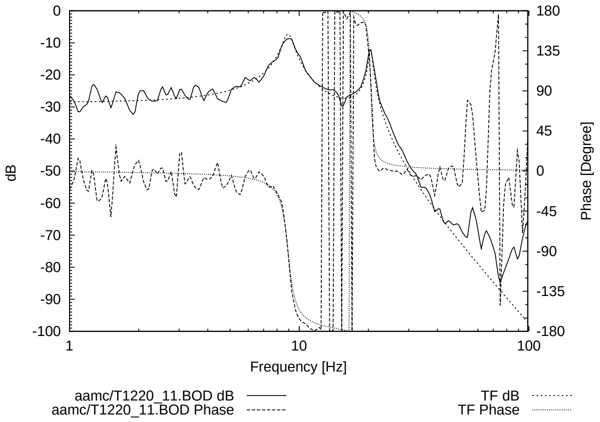
```
pole0:f = 8.688076
pole0:q = 9.340454
pole1:f = 20.08673
pole1:q = 16.34634
```

6.4 Output Mirror Pitch PZT ("m2ypzt")



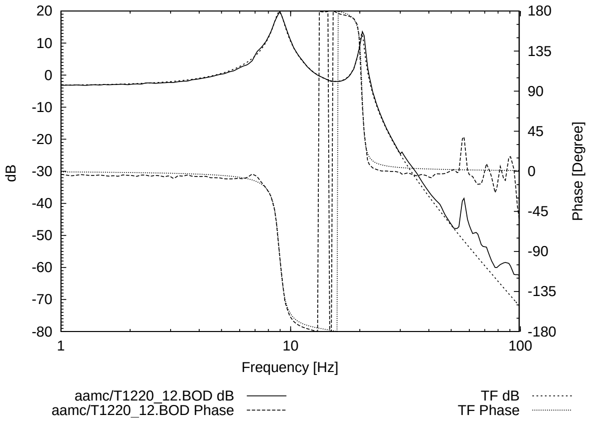
```
Pi Pitch error       T1220_10.BOD

pole0:f = 8.9312815 +- 8.936m (0.1%)
pole0:q = 11.488715 +- 297.2m (2.59%)
pole1:f = 20.561491 +- 19.32m (0.094%)
pole1:q = 29.210305 +- 1.681 (5.76%)
factor = -110.87858m +- -1.238m (1.12%)
```



```
Pi/2 Pitch error     T1220_11.BOD

pole0:f = 8.947484 +- 15.86m (0.177%)
pole0:q = 9.3417483 +- 354.6m (3.8%)
pole1:f = 20.507774 +- 30.04m (0.146%)
pole1:q = 27.878288 +- 2.424 (8.69%)
factor = 37.419176m +- 645.2u (1.72%)
```



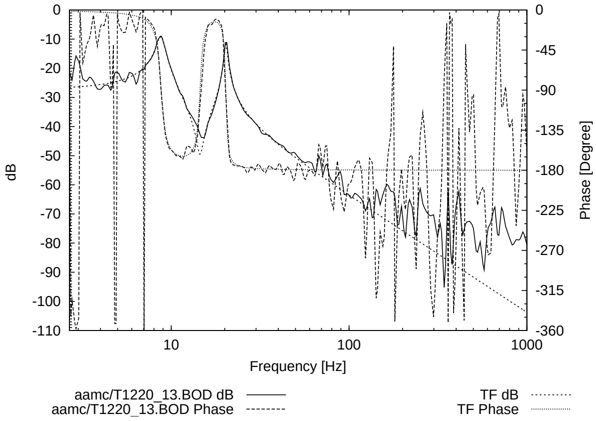
```
End QPD DC Pitch     T1220_12.BOD

pole0:f = 8.9347062 +- 5.266m (0.0589%)
pole0:q = 11.469713 +- 173.4m (1.51%)
pole1:f = 20.448379 +- 11.75m (0.0575%)
pole1:q = 27.958598 +- 926.9m (3.32%)
factor = 688.79131m +- 4.501m (0.653%)
```

Weighted Averages:

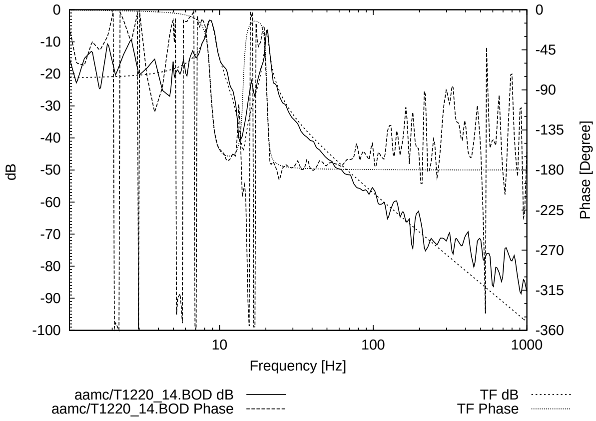
```
pole0:f = 8.934858
pole0:q = 11.15256
pole1:f = 20.48184
pole1:q = 28.21300
```

6.5 Input Mirror Pitch coil (“m1ycoil”)



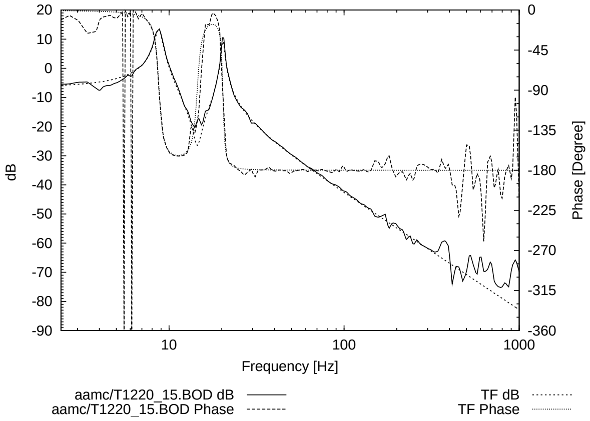
```
Pi Pitch error       T1220_13.BOD

pole0:f = 8.7707005 +- 19.38m (0.221%)
pole0:q = 10.585199 +- 541.9m (5.12%)
zero0:f = 14.532192 +- 125.6m (0.864%)
zero0:q = 15.263638 +- 4.742 (31.1%)
pole1:f = 20.291927 +- 33.79m (0.167%)
pole1:q = 27.515739 +- 2.803 (10.2%)
factor = 43.351314m +- 975.7u (2.25%)
```



```
Pi/2 Pitch error     T1220_14.BOD

pole0:f = 8.7581187 +- 23.51m (0.268%)
pole0:q = 10.768086 +- 656.8m (6.1%)
zero0:f = 14.220014 +- 291m (2.05%)
zero0:q =  16.341334 */ 1.856
pole1:f = 20.299641 +- 47.29m (0.233%)
pole1:q = 23.289386 +- 3.274 (14.1%)
factor = 86.457836m +- 3.091m (3.58%)
```



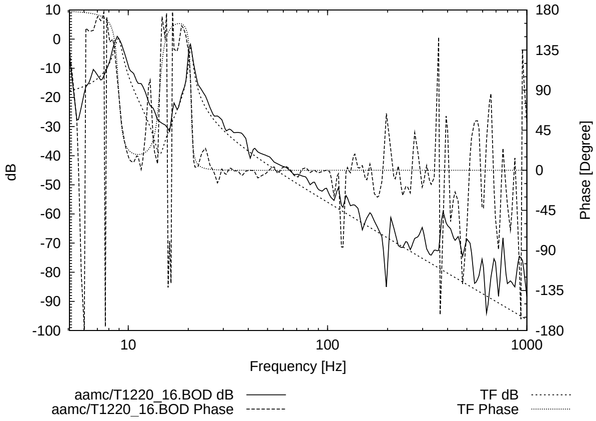
```
End QPD DC Pitch     T1220_15.BOD

pole0:f = 8.7555137 +- 8.536m (0.0975%)
pole0:q = 12.695277 +- 333.3m (2.63%)
zero0:f = 14.497233 +- 67.74m (0.467%)
zero0:q = 11.732884 +- 1.393 (11.9%)
pole1:f = 20.321918 +- 16.99m (0.0836%)
pole1:q = 28.889025 +- 1.515 (5.24%)
factor = 478.86074m +- 5.221m (1.09%)
```

Weighted Averages:

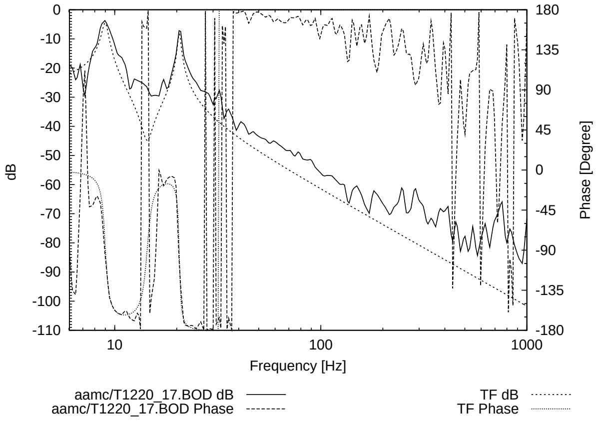
```
pole0:f = 8.757994
pole0:q = 11.90231
pole1:f = 20.31437
pole1:q = 27.83178
zero0:f = 14.49367
zero0:q = 12.04289
```

6.6 Output Mirror Pitch coil ("m2ycoil")



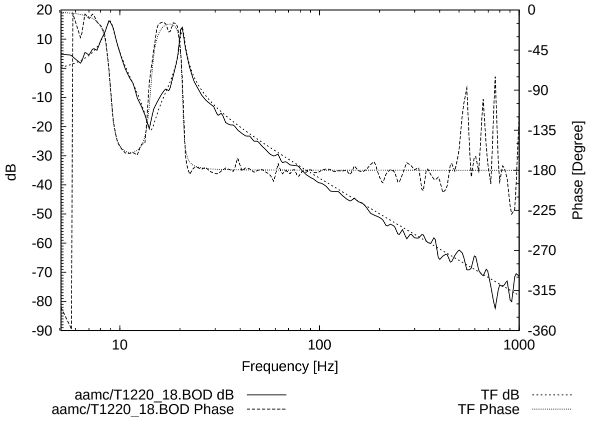
```
Pi Pitch error       T1220_16.BOD

pole0:f = 8.9343651 +- 27.53m (0.308%)
pole0:q = 14.281851 +- 1.225 (8.58%)
zero0:f = 14.399736 +- 235m (1.63%)
zero0:q = 10.829207 +- 272.2m (2.51%) > MAX
pole1:f = 20.60976 +- 44.38m (0.215%)
pole1:q = 33.383364 +- 5.186 (15.5%)
factor = -94.29963m +- -3.853m (4.09%)
```



```
Pi/2 Pitch error     T1220_17.BOD

pole0:f = 8.9970043 +- 32.66m (0.363%)
pole0:q = 15.935303 +- 1.807 (11.3%)
zero0:f = 14.374722 +- 317.3m (2.21%)
zero0:q = 10.838184 +- 241.2m (2.23%) > MAX
pole1:f = 20.577195 +- 61.19m (0.297%)
pole1:q = 33.687723 +- 6.336 (18.8%)
factor = 49.476234m +- 2.697m (5.45%)
```



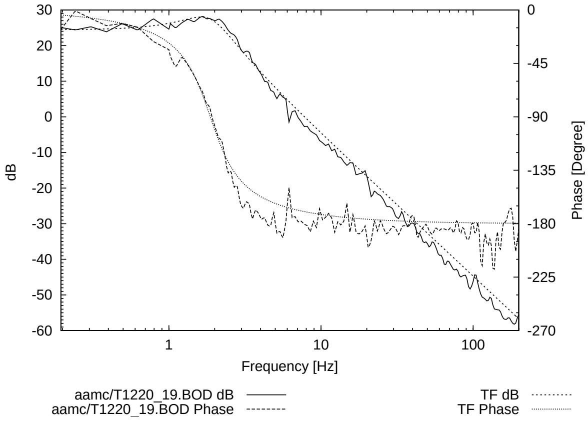
```
End QPD DC Pitch     T1220_18.BOD

pole0:f = 8.9471133 +- 10.88m (0.122%)
pole0:q = 11.299087 +- 344.5m (3.05%)
zero0:f = 14.280006 +- 68.37m (0.479%)
zero0:q = 10.79691 +- 1.226 (11.4%)
pole1:f = 20.579641 +- 17.91m (0.087%)
pole1:q = 26.245019 +- 1.311 (4.99%)
factor = 761.14975m +- 10.98m (1.44%)
```

Weighted Averages:

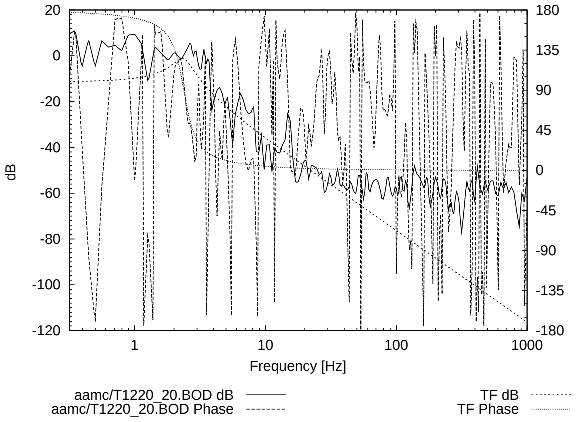
```
pole0:f = 8.949924
pole0:q = 11.66273
pole1:f = 20.58340
pole1:q = 26.94568
zero0:f = 14.29287
zero0:q = 10.79691
```

6.7 End Mirror Yaw Coil (“m3xcoil”)



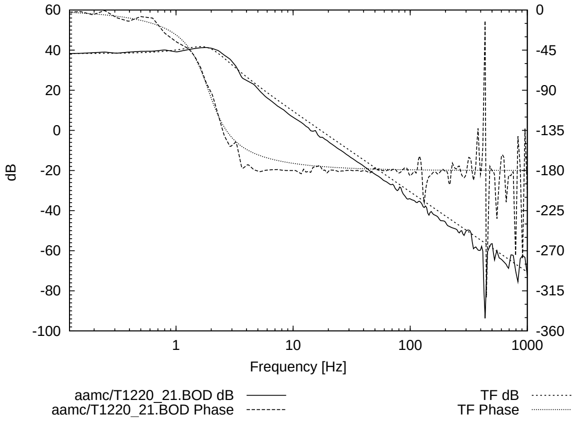
Pi Yaw error   T1220_19.BOD

pole0:f = 1.8713374 +- 11.13m (0.595%)
pole0:q = 1.4166323 +- 29.77m (2.1%)
factor = 16.61447 +- 181.6m (1.09%)



Pi/2 Coil Yaw error T1220_20.BOD

pole0:f = 2.3769774 +- 87.34m (3.67%)
pole0:q = 3.1497205 +- 1.04 (33%)
factor = -270.8383m +- -49.8m (18.4%)



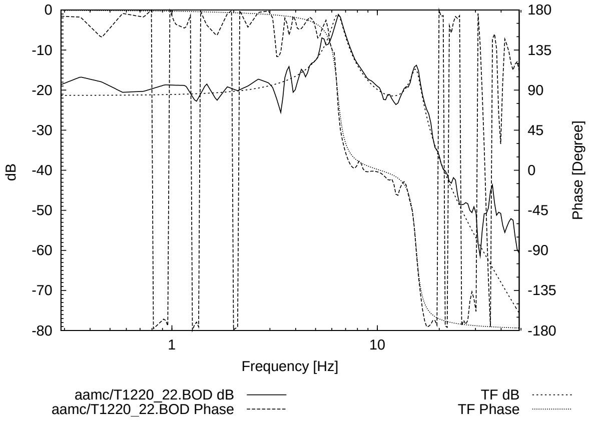
End QPD DC Yaw   T1220_21.BOD

pole0:f = 1.8896638 +- 8.812m (0.466%)
pole0:q = 1.3891876 +- 22.04m (1.59%)
factor = 82.249734 +- 641.4m (0.78%)

Weighted Averages:

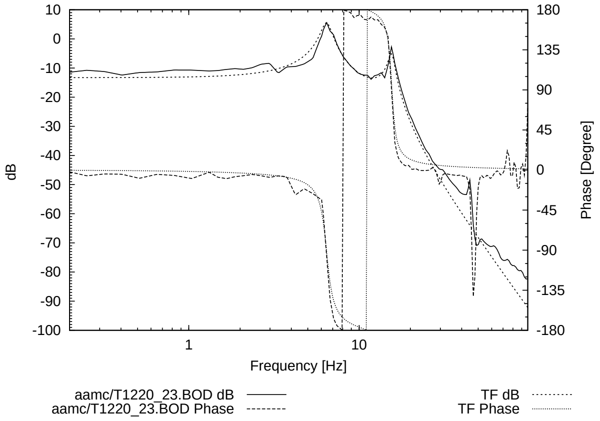
pole0:f = 1.8826161
pole0:q = 1.3989960

6.8 End Mirror Pitch PZT (“m3ypzt”)



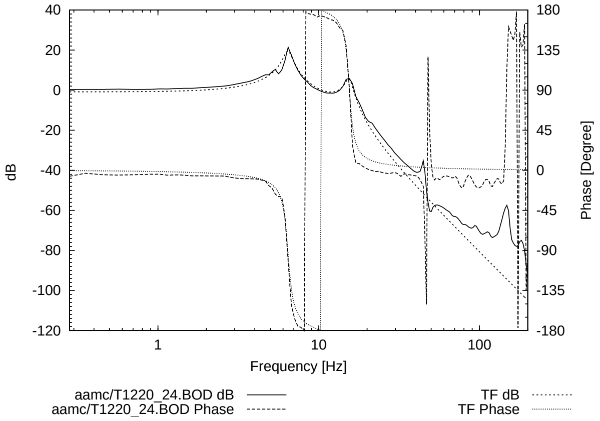
```
Pi Pitch error       T1220_22.BOD

pole0:f = 6.4108286 +- 18.1m (0.282%)
pole0:q = 8.4989131 +- 459.5m (5.41%)
pole1:f = 15.438905 +- 94.27m (0.611%)
pole1:q = 10.134435 +- 1.324 (13.1%)
factor = -85.75237m +- -2.02m (2.36%)
```



```
Pi/2 Pitch error     T1220_23.BOD

pole0:f = 6.4501997 +- 17.03m (0.264%)
pole0:q = 7.2412406 +- 310.6m (4.29%)
pole1:f = 15.548908 +- 63.98m (0.411%)
pole1:q = 12.259952 +- 1.299 (10.6%)
factor = 216.30985m +- 4.018m (1.86%)
```



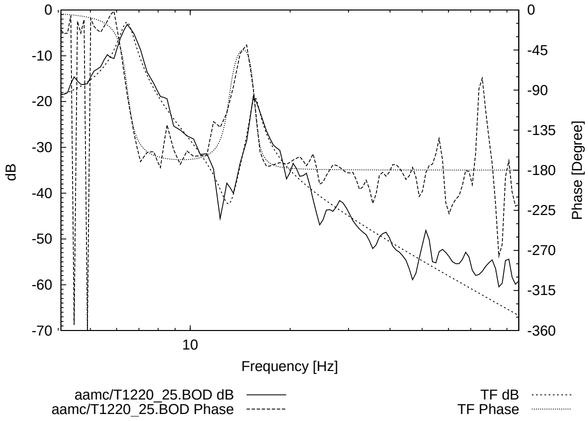
```
End QPD DC Pitch     T1220_24.BOD

pole0:f = 6.441543 +- 11.96m (0.186%)
pole0:q = 8.6539705 +- 308.9m (3.57%)
pole1:f = 15.47822 +- 62.81m (0.406%)
pole1:q = 10.059454 +- 849.4m (8.44%)
factor = 904.141m +- 14m (1.55%)
```

Weighted Averages:

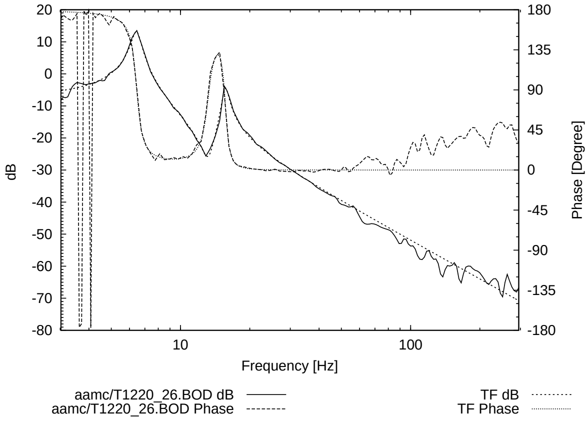
```
pole0:f = 6.436784
pole0:q = 8.052757
pole1:f = 15.49933
pole1:q = 10.58777
```

6.9 End Mirror Pitch coil ("m3ycoil")



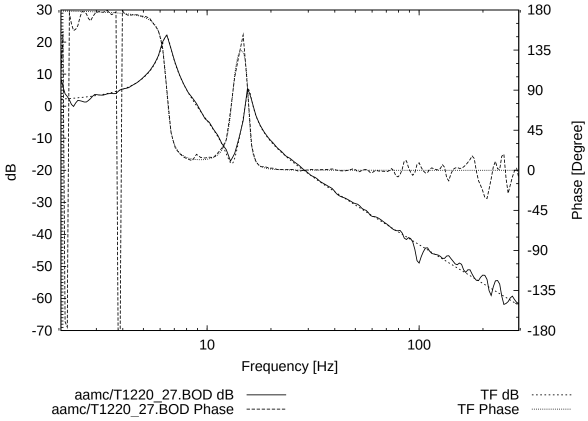
```
Pi Pitch error        T1220_25.BOD

pole0:f = 6.4562745 +- 12.24m (0.19%)
pole0:q = 10.880457 +- 519.8m (4.78%)
zero0:f = 13.118087 +- 141.4m (1.08%)
zero0:q = 10.177679 +- 2.379 (23.4%)
pole1:f = 15.621772 +- 66.15m (0.423%)
pole1:q = 16.829252 +- 2.755 (16.4%)
factor = 73.69469m +- 1.956m (2.65%)
```



```
Pi/2 Pitch error      T1220_26.BOD

pole0:f = 6.4394746 +- 4.254m (0.0661%)
pole0:q = 11.47595 +- 191.8m (1.67%)
zero0:f = 13.214751 +- 53.88m (0.408%)
zero0:q = 8.8553113 +- 623m (7.04%)
pole1:f = 15.652186 +- 26.16m (0.167%)
pole1:q = 15.990668 +- 937.6m (5.86%)
factor = -433.8639m +- -3.66m (0.843%)
```



```
End QPD DC Pitch      T1220_27.BOD

pole0:f = 6.4356973 +- 6.681m (0.104%)
pole0:q = 11.732925 +- 311.2m (2.65%)
zero0:f = 13.167754 +- 85.29m (0.648%)
zero0:q = 9.613809 +- 1.201 (12.5%)
pole1:f = 15.663831 +- 37.03m (0.236%)
pole1:q = 17.752179 +- 1.652 (9.31%)
factor = -1.1676377 +- -14.64m (1.25%)
```

Weighted Averages:

```
pole0:f = 6.439799
pole0:q = 11.48701
pole1:f = 15.65282
pole1:q = 16.45207
zero0:f = 13.19353
zero0:q = 9.075651
```

7 Calibration Factors

Both actuators and sensors have calibration factors which are not yet known. In order to find them (and test the above theroretical model), the LISO-fitted `factor`s from the transfer functions were fitted to simple model

$$\texttt{factor} = A_{\text{optical}} \times G_{\text{actuator}} \times G_{\text{sensor}}. \tag{19}$$

where *all* measurements were fitted simultaneously to one set of calibration constants G_i.

For that purpose a simple C program ("`prod.c`") was written which uses the Nelder-Mead Simplex algorithm to minimize

$$\chi^2(G_1, \ldots, G_m) = \sum_{i=1}^{n_{\text{data}}} w_i \left| f_i - \tilde{f}_i(G_1, \ldots, G_m) \right|^2 \tag{20}$$

where f_i are the measured factors, $\tilde{f}_i$ their approximations from Equation 19, $w_i = 1/\sigma_i^2$ are the weights of each data point derived from their standard deviation (taken from the LISO fit), and G_i the unknown calibration constants.

As input the following data were used:

$f_{\text{meas.}}$	σ_i	A_{optical}	G_{actuator}	G_{sensor}
5.0043609	40.31×10^{-3}	-1.019	`m1xcoil`	`xi`
3.2644629	29.84×10^{-3}	1.366	`m1xcoil`	`xq`
2.7204019	63.32×10^{-3}	$-.3383$	`m1xcoil`	`xe`
2.4554303	33.68×10^{-3}	-1.019	`m2xcoil`	`xi`
-3.5256953	21.65×10^{-3}	-1.366	`m2xcoil`	`xq`
-2.1236194	32.95×10^{-3}	$.2454$	`m2xcoil`	`xe`
62.862325×10^{-3}	1.561×10^{-3}	0.7035	`m1ypzt`	`yi`
84.273794×10^{-3}	1.359×10^{-3}	-0.524	`m1ypzt`	`yq`
590.1168×10^{-3}	7.742×10^{-3}	-1.701	`m1ypzt`	`ye`
$-110.87858 \times 10^{-3}$	1.238×10^{-3}	-0.7035	`m2ypzt`	`yi`
37.419176×10^{-3}	645.2×10^{-6}	-0.524	`m2ypzt`	`yq`
688.79131×10^{-3}	4.501×10^{-3}	-1.705	`m2ypzt`	`ye`
43.351314×10^{-3}	975.7×10^{-6}	0.7035	`m1ycoil`	`yi`
86.457836×10^{-3}	3.091×10^{-3}	-0.524	`m1ycoil`	`yq`
478.86074×10^{-3}	5.221×10^{-3}	-1.701	`m1ycoil`	`ye`
-94.29963×10^{-3}	3.853×10^{-3}	-0.7035	`m2ycoil`	`yi`
49.476234×10^{-3}	2.697×10^{-3}	-0.524	`m2ycoil`	`yq`
761.14975×10^{-3}	10.98×10^{-3}	-1.705	`m2ycoil`	`ye`
16.61447	181.6×10^{-3}	2.906	`m3xcoil`	`xi`
-270.8383×10^{-3}	49.8×10^{-3}	0	`m3xcoil`	`xq` $* **$
82.249734	641.4×10^{-3}	4.8613	`m3xcoil`	`xe`
-85.75237×10^{-3}	2.02×10^{-3}	0	`m3ypzt`	`yi` $* **$
216.30985×10^{-3}	4.018×10^{-3}	-2.105	`m3ypzt`	`yq`
904.141×10^{-3}	14×10^{-3}	-2.105	`m3ypzt`	`ye`
73.69469×10^{-3}	1.956×10^{-3}	$0.$	`m3ycoil`	`yi` $* **$
-433.8639×10^{-3}	3.66×10^{-3}	-2.105	`m3ycoil`	`yq`
-1.1676377	14.64×10^{-3}	-2.105	`m3ycoil`	`ye`

(21)

The lines marked `***` at the end were excluded from the fit, because the theoretically expected factor was zero (and the measured one was indeed small).

The resulting calibration constants are:

$$
\begin{aligned}
\texttt{xi} &= 0.339034\\
\texttt{xq} &= -0.256563\\
\texttt{yi} &= -0.361489\\
\texttt{yq} &= 0.326227\\
\texttt{m1xcoil} &= -10.3098\\
\texttt{m2xcoil} &= -8.37122\\
\texttt{m1ycoil} &= -0.2823\\
\texttt{m2ycoil} &= -0.44523\\
\texttt{m1ypzt} &= -0.346921\\
\texttt{m2ypzt} &= -0.40358\\
\texttt{m3xcoil} &= 16.9186\\
\texttt{m3ypzt} &= -0.425773\\
\texttt{m3ycoil} &= 0.556748
\end{aligned}
\tag{22}
$$

The residuals were:

$f_{\text{meas.}}$	σ_i	$\tilde{f}$	ratio	ratio
5.00436	0.04031	3.5618	1.40501	2.95 dB
3.26446	0.02984	3.61324	0.903472	−0.88 dB
2.7204	0.06332	3.48782	0.779972	−2.16 dB
2.45543	0.03368	2.89205	0.849027	−1.42 dB
−3.5257	0.02165	−2.93382	1.20174	1.60 dB
−2.12362	0.03295	−2.0543	1.03374	0.29 dB
0.0628623	0.001561	0.0882244	0.712527	−2.94 dB
0.0842738	0.001359	0.0593036	1.42106	3.05 dB
0.590117	0.007742	0.590112	1.00001	0.00 dB
−0.110879	0.001238	−0.102633	1.08034	0.67 dB
0.0374192	0.0006452	0.0689892	0.542392	−5.31 dB
0.688791	0.004501	0.688104	1.001	0.01 dB
0.0433513	0.0009757	0.0717908	0.603856	−4.38 dB
0.0864578	0.003091	0.0482571	1.79161	5.06 dB
0.478861	0.005221	0.480192	0.997228	−0.02 dB
−0.0942996	0.003853	−0.113225	0.83285	−1.59 dB
0.0494762	0.002697	0.076109	0.650071	−3.74 dB
0.76115	0.01098	0.759117	1.00268	0.02 dB
16.6145	0.1816	16.6688	0.99674	−0.03 dB
82.2497	0.6414	82.2466	1.00004	0.00 dB
0.21631	0.004018	0.292382	0.73982	−2.62 dB
0.904141	0.014	0.896252	1.0088	0.08 dB
−0.433864	0.00366	−0.382323	1.13481	1.10 dB
−1.16764	0.01464	−1.17195	0.996316	−0.03 dB

(23)