à Guoy shift telescope for TAMA300 main cavity alignment ~ Front Mirror

Original by Gerhard Heinzel
Interpreted by K. Arai (2001/5/6)

Ÿ Constant

```
In[131]:= H* Nd:YAG wavelength *L
          l = 1064.*^-9;
```

Ÿ Rayleigh range

```
In[132]:= H* Radius of curvature at z *L
          R@z_D := z + zr^2 • z

In[133]:= zr =.
          zr = zr /. Solve@R@300D Š 450, zrD@@2DD

Out[134]= 150 • Â2
```

Ÿ q parameter at waist (q0)

```
In[135]:= q0 = I zr

Out[135]= 150 ä • Â2

In[136]:= H* q=z+q0 *L
```

Ÿ Beam diameter in mm

```
In[137]:= H* From 1/q = 1/R + I l/(p wHzL^2)  *L

In[138]:= w@q_D := 2 1000 SqrtA-AbsA 1/Im@1/qDE/Pi E

In[139]:= H* Plot@w@z+q0D,8z,0,1000<D *L
```

Ÿ Guoy Shift between 00 and 01 modes in degrees

```
In[140]:= guoy@q_D := 180/p ArcTanA Re@qD/Im@qDE
```

Ÿ ABCD matrices

Transform q1 -> q2 by ABCD matrix (M=J $\begin{matrix} A & B \\ C & D \end{matrix}$ N)

```
In[141]:= transf@q1_, M_D := HM@@1, 1DD q1 + M@@1, 2DDL • HM@@2, 1DD q1 + M@@2, 2DDL
```

ABCD Matrix for lens of focal length f

```
In[142]:= lens@f_D := 881, 0<, 8-1 • f, 1<<
```

ABCD Matrix for vacuum distance d

```
In[143]:= dist@d_D := 881, d<, 80, 1<<
```

Ÿ q's and guoy shifts at each optical component

Distance from waist (front mirror) to lens1

```
In[144]:= d1 = 4.;
```

q and guoy shift just before lens1
dg1 is a phase shift between the waist and lens1

```
In[145]:= q1 := transf@q0, dist@d1DD
          dg1 := guoy@q1D
```

q just after lens1

```
In[147]:= q2 := transf@q1, lens@f1DD
```

q and guoy shift just before lens2 as a function of d2
d2 is a distance between lens1 and lens2
dg2 is a phase shift between lens1 and lens2

```
In[148]:= q3@d2_D := transf@q2, dist@d2DD
          dg2@d2_D := guoy@q3@d2DD - guoy@q2D
```

q just after lens2

```
In[150]:= q4@d2_D := transf@q3@d2D, lens@f2DD
```

q and guoy shift just before PD as a function of d2 and d3
d3 is a distance between lens2 and PD
dg3 is a phase shift between lens2 and PD

```
In[151]:= q5@d2_, d3_D := transf@q4@d2D, dist@d3DD
          dg3@d2_, d3_D := guoy@q5@d2, d3DD - guoy@q4@d2DD
```

Total quoy phase shift from the waist to PD

```
In[153]:= dg@d2_, d3_D := dg1 + dg2@d2D + dg3@d2, d3D
            H* use 180 deg to see how much the shift above 180 deg is *L
```

Beam diameter at PD

```
In[154]:= size@d2_, d3_D := w@q5@d2, d3DD
```

Ÿ Lens focal length & position

```
In[155]:= f1 = 2.;H*Focal Length First Lens*L
          d2 = 2.55;H*First Lens-Second Lens*L
          f2 = 1;H*Focal Length Second Lens*L
          d3 = 0.592;H*Second Lens-Second Detector*L
```

Ÿ Reporting parameters

```
In[159]:= Print@"Rayleigh Range               ", N@zrD, " m"D;
          Print@"lambda                       ", l, " m"D;
          Print@"d1 Hwaist - first lensL      ", d1, " m"D;
          Print@"d2 Hfirst lens - second lensL", d2, " m"D;
          Print@"d3 Hsecond lens - detectorL  ", d3, " m"D;
          Print@"f1 Hfocal length first lensL ", f1, " m"D;
          Print@"f2 Hfocal length first lensL ", f2, " m"D;
          Print@"Design beam diamater         ", 3, " mm"D;
          Print@"Design Guoy Shift            ", 180, " Deg."D;
```

Rayleigh Range	212.132 m
lambda	1.064×10^{-6} m
d1 Hwaist - first lensL	4. m
d2 Hfirst lens - second lensL	2.55 m
d3 Hsecond lens - detectorL	0.592 m
f1 Hfocal length first lensL	2. m
f2 Hfocal length first lensL	1 m
Design beam diamater	3 mm
Design Guoy Shift	180 Deg.

Ÿ Find PD position

```
In[168]:= d3 =.;
          d3 = d3 /. Chop@FindRoot@dg@d2, d3D Š 180, 8d3, 0, 1<DD
Out[169]= 0.591837
```

```
In[170]:= Print@"d1=", d1, " f1=", f1, " d2=", d2, " f2=", f2, " d3=", d3D ;
```

```
d1=4. f1=2. d2=2.55 f2=1 d3=0.591837
```

```
In[171]:= Print@"dg=", dg@d2, d3D, " size=", size@d2, d3DD
```

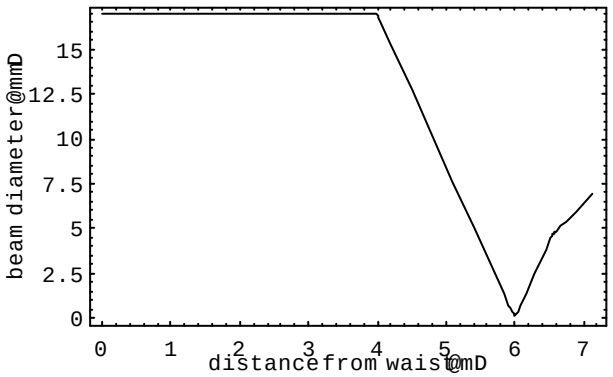
```
dg=180. size=6.91931
```

Ÿ Combine Functions for plotting

```
In[172]:= wx@x_, d2_, d3_D := w@q0 + xD •; x £ d1
          wx@x_, d2_, d3_D := w@q2 + x - d1D •; x > d1 && x £ d1 + d2
          wx@x_, d2_, d3_D := w@q4@d2D + x - d1 - d2D •; x > d1 + d2
          gx@x_, d2_, d3_D := guoy@q0 + xD •; x £ d1
          gx@x_, d2_, d3_D := dg1 + guoy@q2 + x - d1D - guoy@q2D •; x > d1 && x £ d1 + d2
          gx@x_, d2_, d3_D := dg1 + dg2@d2D + guoy@q4@d2D + x - d1 - d2D - guoy@q4@d2DD •; x > d1 + d2
```

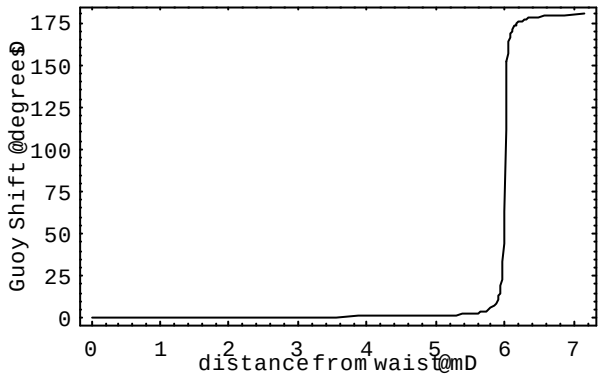
Ÿ Plots

```
In[178]:= p1 =
            Plot@
             wx@x, d2, d3D,
             8x, 0, d1 + d2 + d3<,
             FrameLabel ® 8
               "distance from waist@mD",
               "beam diameter @mmD"
              <,
             Frame ® True
            D;
```

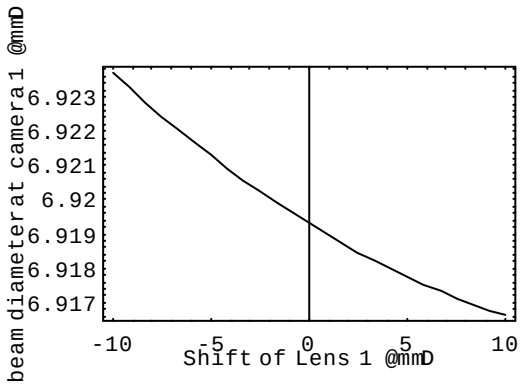



```
In[179]:= H*lens2 0.55 m away from the second waist*L
```

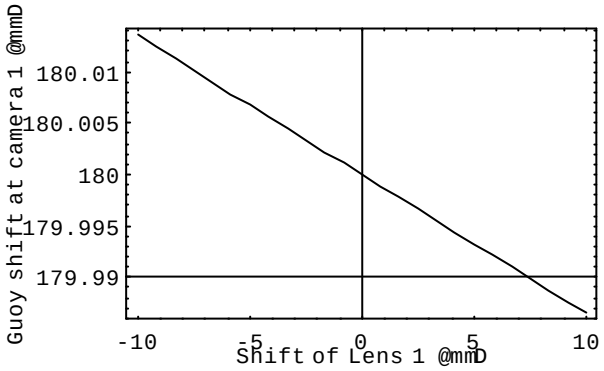
In[180]:=
```
p2 =
  Plot@
   gx@x, d2, d3D,
   8x, 0, d1 + d2 + d3<,
   FrameLabel ® 8
    "distance from waist@mD",
    "Guoy Shift @degreesD"
    <,
   Frame ® True
  D;
```



In[181]:=
```
p3 =
  Plot@
   size@d2 + shift • 1000, d3 - shift • 1000D,
   8shift, -10, 10<,
   FrameLabel ® 8
     "Shift of Lens 1 @mmD",
    "beam diameter at camera 1 @mmD"
    <,
   Frame ® True
  D;
```



```
In[182]:= p4 =
            Plot@
             dg@d2 + shift • 1000, d3 - shift • 1000D,
             8shift, -10, 10<,
             FrameLabel ® 8
               "Shift of Lens 1 @mmD",
               "Guoy shift at camera 1 @mmD"
              <,
             Frame ® True
            D;
```



à Guoy shift telescope for TAMA300 main cavity alignment ~ End Mirror

Original by Gerhard Heinzel
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Ÿ Constant

```
In[183]:= H* Nd:YAG wavelength *L
          l = 1064.*^-9 ;
```

Ÿ Rayleigh range

```
In[184]:= H* Radius of curvature at z *L
          R@z_D := z + zr^2 • z

In[185]:= zr =.
          zr = zr /. Solve@R@300D Š 450, zrD@@2DD

Out[186]= 150 • 2
```

Ÿ q parameter at waist (q0)

```
In[187]:= q0 = I zr
```

```
Out[187]= 150 ä • ∫ 2
```

```
In[188]:= H* q=z+q0 *L
```

Ÿ Beam diameter in mm

Ÿ Guoy Shift between 00 and 01 modes in degrees

```
In[192]:= guoy@q_D := 180/p ArcTanA Re@qD/Im@qD E
```

Ÿ ABCD matrices

Transform q1 -> q2 by ABCD matrix (M=J A B / C D N)

```
In[193]:= transf@q1_, M_D := HM@@1, 1DD q1 + M@@1, 2DDL • HM@@2, 1DD q1 + M@@2, 2DDL
```

ABCD Matrix for lens of focal length f

```
In[194]:= lens@f_D := 881, 0<, 8-1 • f, 1<<
```

ABCD Matrix for vacuum distance d

```
In[195]:= dist@d_D := 881, d<, 80, 1<<
```

Ÿ q's and guoy shifts at each optical component

Distance from waist (front mirror) to lens1

```
In[196]:= d1 = 4.;
```

q and guoy shift just before lens1
dg1 is a phase shift between the waist and lens1

```
In[197]:= q1 := transf@q0, dist@d1DD
          dg1 := guoy@q1D
```

q just after lens1

```
In[199]:= q2 := transf@q1, lens@f1DD
```

q and guoy shift just before lens2 as a function of d2
d2 is a distance between lens1 and lens2
dg2 is a phase shift between lens1 and lens2

```
In[200]:= q3@d2_D := transf@q2, dist@d2DD
          dg2@d2_D := guoy@q3@d2DD - guoy@q2D
```

q just after lens2

```
In[202]:= q4@d2_D := transf@q3@d2D, lens@f2DD
```

q and guoy shift just before PD as a function of d2 and d3
d3 is a distance between lens2 and PD
dg3 is a phase shift between lens2 and PD

```
In[203]:= q5@d2_, d3_D := transf@q4@d2D, dist@d3DD
          dg3@d2_, d3_D := guoy@q5@d2, d3DD - guoy@q4@d2DD
```

Total quoy phase shift from the waist to PD

```
In[205]:= dg@d2_, d3_D := Mod@dg1 + dg2@d2D + dg3@d2, d3D, 180D
           H* use 180 deg to see how much the shift above 180 deg is *L
```

Beam diameter at PD

```
In[206]:= size@d2_, d3_D := w@q5@d2, d3DD
```

Ÿ Lens focal length & position

```
In[207]:= f1 = 2.;H*Focal Length First Lens*L
          d2 = 2.1998;H*First Lens-Second Lens*L
          f2 = .2;H*Focal Length Second Lens*L
          d3 = 3.25;H*Second Lens-Second Detector*L
```

Ÿ Reporting parameters

```
In[211]:= Print@"Rayleigh Range                ", N@zrD, " m"D;
          Print@"lambda                        ", l, " m"D;
          Print@"d1 Hwaist - first lensL       ", d1, " m"D;
          Print@"d2 Hfirst lens - second lensL ", d2, " m"D;
          Print@"d3 Hsecond lens - detectorL   ", d3, " m"D;
          Print@"f1 Hfocal length first lensL  ", f1, " m"D;
          Print@"f2 Hfocal length first lensL  ", f2, " m"D;
          Print@"Design beam diamater          ", 3, " mm"D;
          Print@"Design Guoy Shift             ", 55, " Deg."D;
```

```
Rayleigh Range                212.132 m
lambda                        1.064 ´ 10^-6 m
d1 Hwaist - first lensL       4. m
d2 Hfirst lens - second lensL 2.1998 m
d3 Hsecond lens - detectorL   3.25 m
f1 Hfocal length first lensL  2. m
f2 Hfocal length first lensL  0.2 m
Design beam diamater          3 mm
Design Guoy Shift             55 Deg.
```

Ÿ Find lens2 position

```
In[220]:= d2 = d2 /. Chop@FindRoot@dg@d2, d3D Š 55., 8d2, f1 + f2 - .02, f1 + f2 + .01<DD
Out[220]= 2.19982
In[221]:= Print@"d1=", d1, " f1=", f1, " d2=", d2, " f2=", f2, " d3=", d3D ;
          d1=4. f1=2. d2=2.19982 f2=0.2 d3=3.25
In[222]:= Print@"dg=", dg@d2, d3D, " size=", size@d2, d3DD
          dg=55. size=2.99526
```

Ÿ Combine Functions for plotting

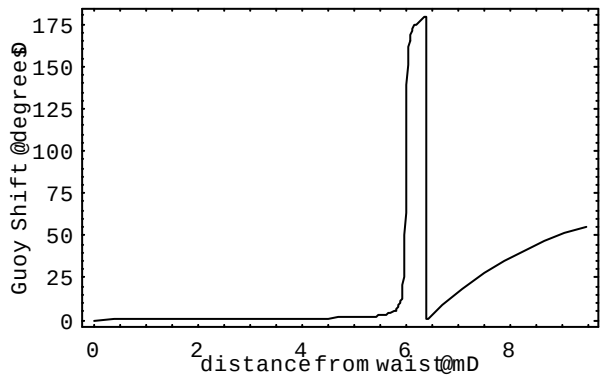
```
In[223]:= wx@x_, d2_, d3_D := w@q0 + xD /; x £ d1
          wx@x_, d2_, d3_D := w@q2 + x - d1D /; x > d1 && x £ d1 + d2
          wx@x_, d2_, d3_D := w@q4@d2D + x - d1 - d2D /; x > d1 + d2
          gx@x_, d2_, d3_D := guoy@q0 + xD /; x £ d1
          gx@x_, d2_, d3_D := dg1 + guoy@q2 + x - d1D - guoy@q2D /; x > d1 && x £ d1 + d2
          gx@x_, d2_, d3_D := dg1 + dg2@d2D + guoy@q4@d2D + x - d1 - d2D - guoy@q4@d2DD /; x > d1 + d2
```

Ÿ Plots

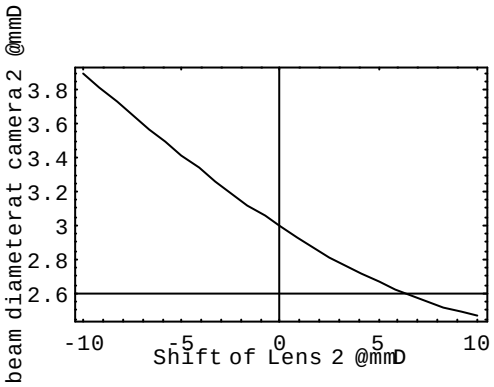
In[229]:=
```
p1 =
  Plot@
   wx@x, d2, d3D,
   8x, 0, d1 + d2 + d3<,
   FrameLabel ® 8
     "distance from waist@mD",
     "beam diameter @mmD"
    <,
   Frame ® True
  D;
```

In[230]:=
```
H* lens at 0.2 m away from the second waist*L
```

In[231]:=
```
p2 =
  Plot@
   Mod@
    gx@x, d2, d3D, 180D,
   8x, 0, d1 + d2 + d3<,
   FrameLabel ® 8
    "distance from waist@mD",
    "Guoy Shift @degreesD"
    <,
   Frame ® True
  D;
```



In[234]:=
```
p3 =
 Plot@
  size@d2 + shift • 1000, d3 - shift • 1000D,
  8shift, -10, 10<,
  FrameLabel ® 8
    "Shift of Lens 2 @mmD",
   "beam diameter at camera 2 @mmD"
   <,
  Frame ® True
 D;
```



In[235]:=
```
p4 =
 Plot@
  dg@d2 + shift • 1000, d3 - shift • 1000D,
  8shift, -10, 10<,
  FrameLabel ® 8
    "Shift of Lens 2 @mmD",
    "Guoy shift at camera 2 @mmD"
   <,
  Frame ® True
 D;
```