Gravitational wave radiometry; numerical study with GPGPU

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Gravitational Wave Radiometry

Strong benefits

- No need to know the target's GW wave form
- Possible to detect the target's direction

Target

- Stochastic GW from celestial objects
- Pulsars and their clusters
- Stochastic GW backgrounds

Conditions

- Use two or more detectors
  > today, only talk about using two detectors case

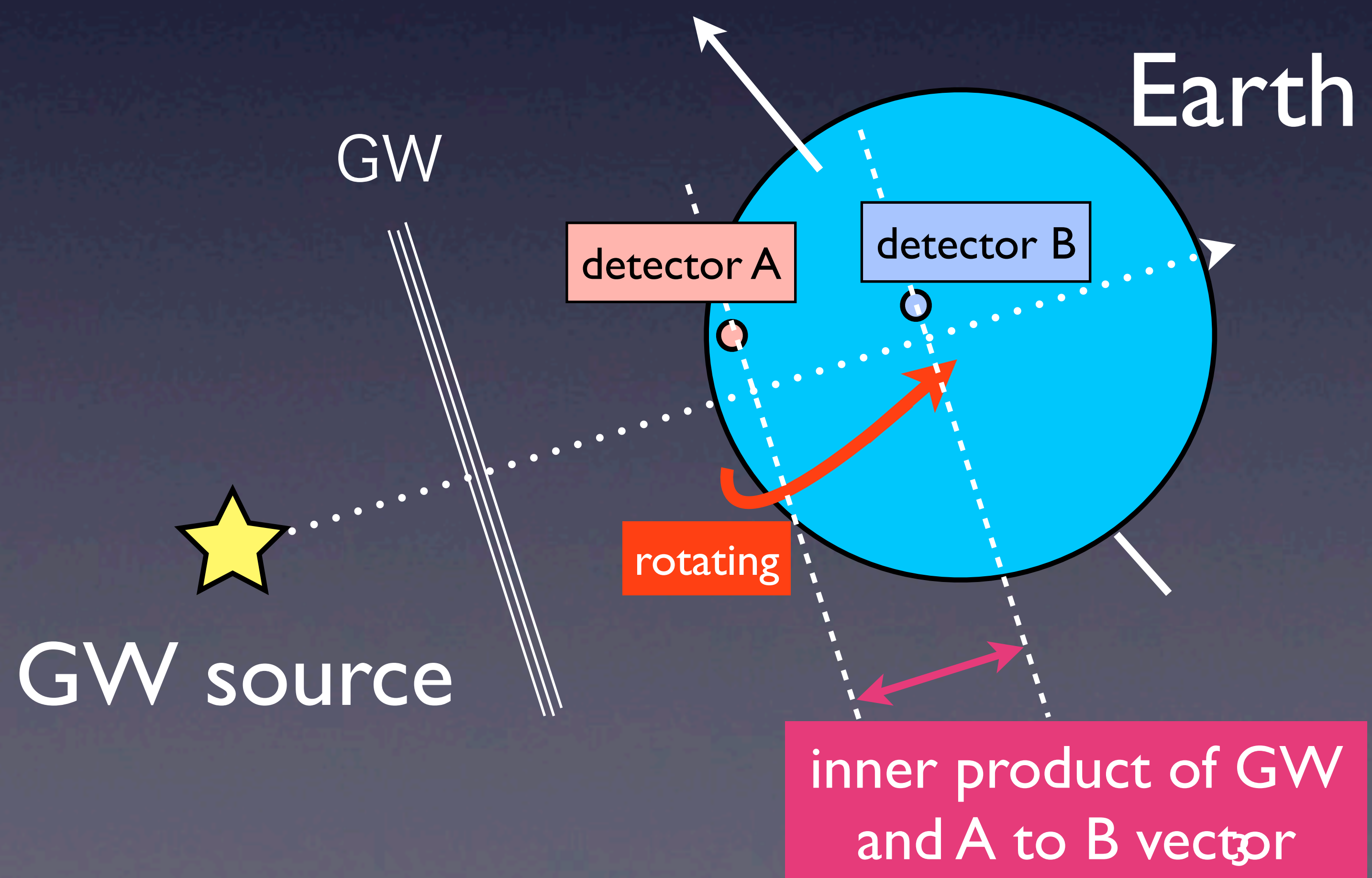
- Necessary to integrate for a long time (for years)

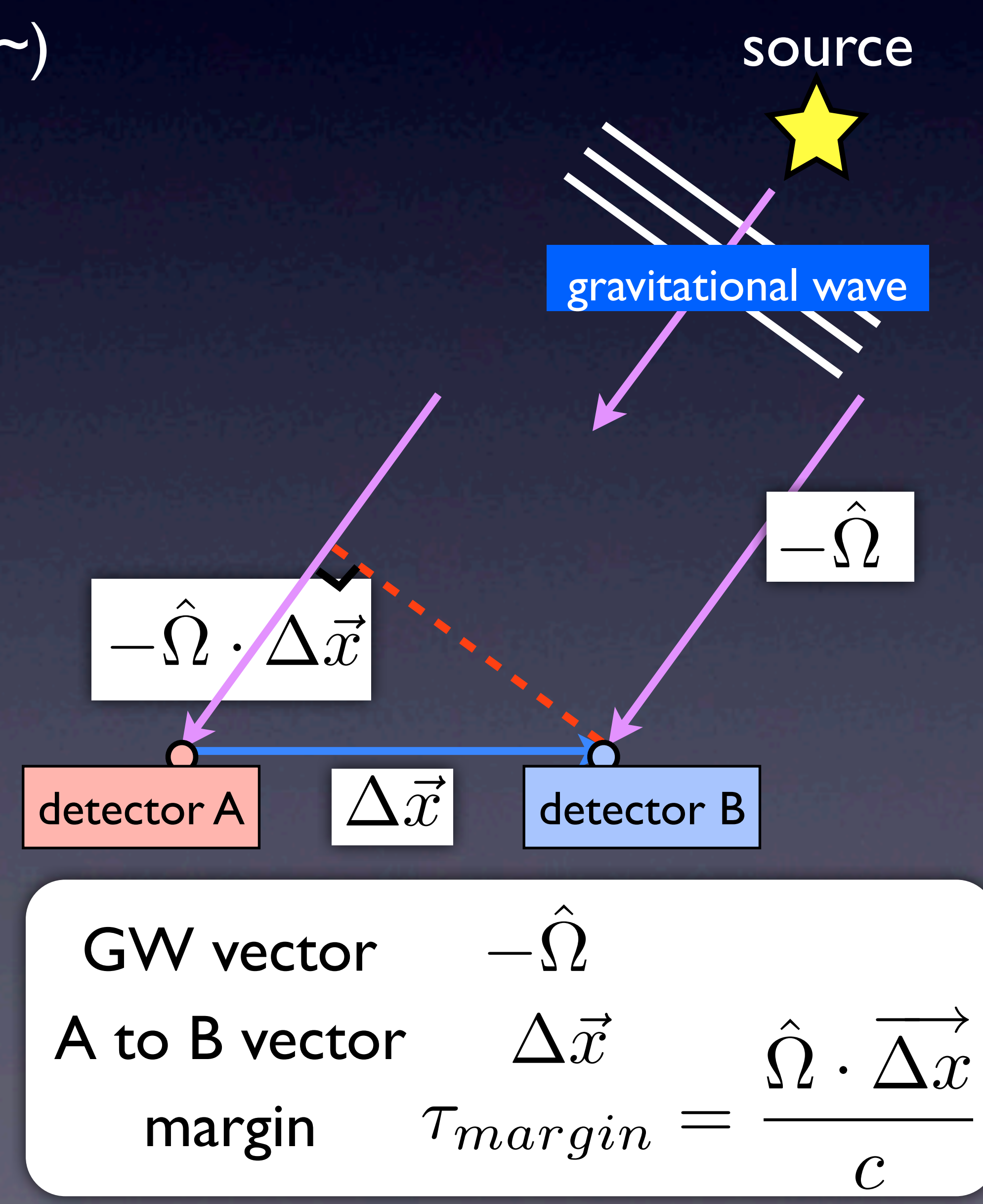
Gravitational Wave Radiometry

theory

- Use two or more detectors on the earth
- Each detectors catch the same GW in deferent time
- By earth's rotation, the deference between the each detected time are modulated
- The time deference can be compensated numerically
- Integrate the compensated data for a long time (for years ~)

➡ Then, we can detect the weak stochastic signals





Gravitational Wave Radiometry

theory

statistics

$$\Delta S(t, \hat{\Omega}) = \int_{-\infty}^{\infty} df \, \tilde{s}_1^*(f;t) \tilde{s}_2(f;t) \tilde{Q}(f, \hat{\Omega}; t)$$

Need two detector's outputs
Calculate each position of the sky

$s_{1,2}(f;t)$ detector's output

$\hat{\Omega}$ position of the sky

$\tilde{Q}(f, \hat{\Omega}; t)$ Radiometry filter function

compensate the time deference here

signal to noise ratio

$$\rho(\hat{\Omega}) = \frac{\mu(\hat{\Omega})}{\sigma(\hat{\Omega})}$$

$$\mu = \langle \Delta S \rangle, \sigma^2 = \left\langle [\Delta S - \mu]^2 \right\rangle$$

The signal to noise ratio is depend on the mean of statistics and that of scattering

$$\Delta S(t,\hat{\Omega}) = \int_{-\infty}^{\infty} df\, \tilde{s}_1^*(f;t)\tilde{s}_2(f;t)\tilde{Q}(f,\hat{\Omega};t)$$

$$\tilde{s}_{1,2}(f;t) = h_{1,2}(f;t) + n_{1,2}(f;t)$$

output — GW signal — noise

$\tilde{Q}(f,\hat{\Omega};t)$ Radiometry filter function compensate the margin

Simplify theory

- GW signal has correlation between two detectors
- Noise and GW, noise and noise has no correlation
- Cross correlation between two detector's GW signal part are maximized by the Q filter

$$\tilde{Q}(f,\hat{\Omega};t) = \lambda \frac{\gamma^*(f,\hat{\Omega})H(f)}{P_1(f)P_2(f)}$$

Q filter

$P_1(f), P_2(f)$ noise spectrum

window function

$$\gamma(f,\hat{\Omega}) = \sum_{A=+,\times} F_1^A F_2^A e^{2\pi f i\hat{\Omega}\cdot\Delta\vec{x}/c}$$

compensated term

$H(f)$ The only part to consider about the GW's frequency characteristics

for unknown wave form

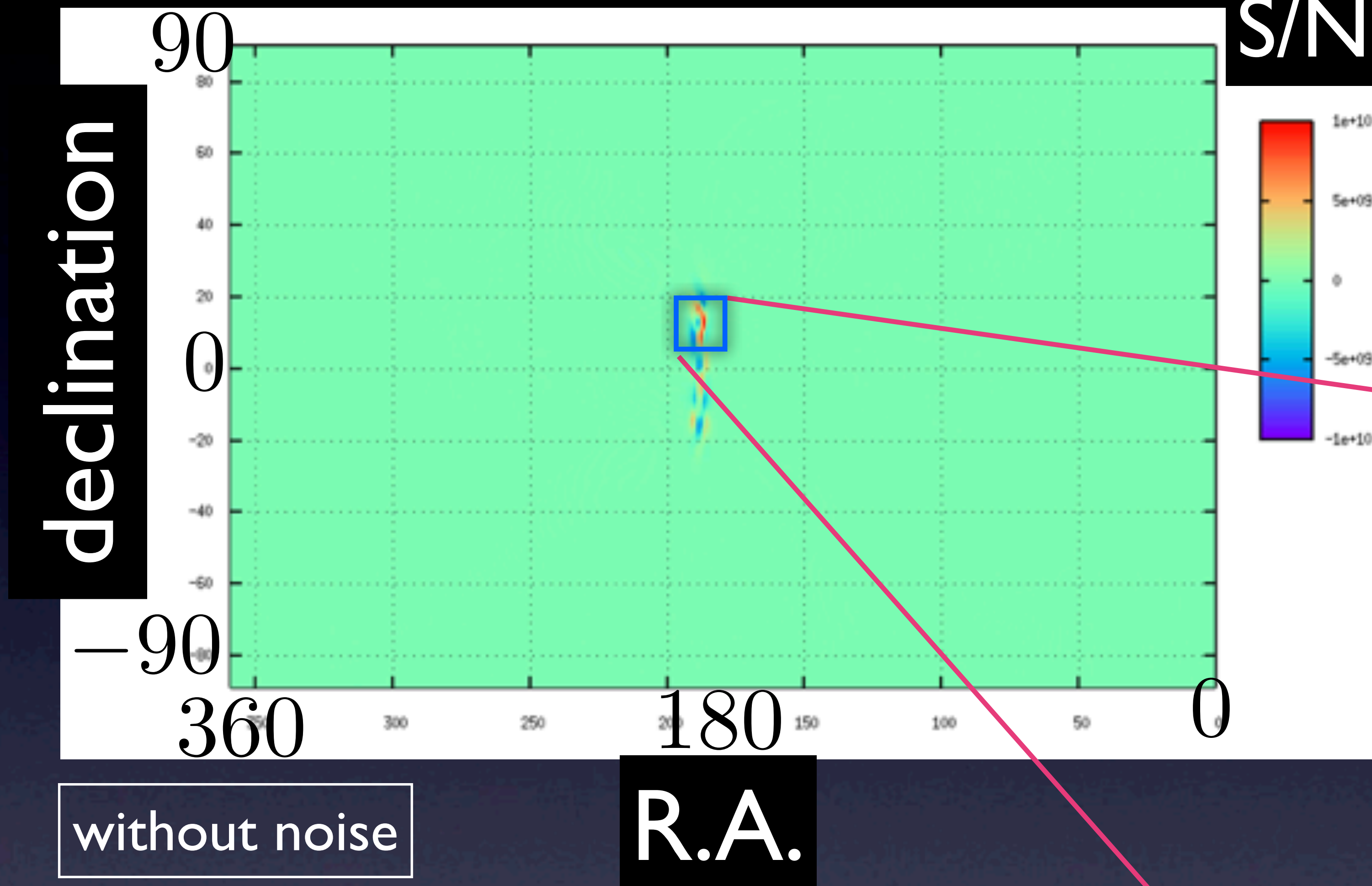
$$H(f) = \begin{cases} 0 (f < low[Hz], f > high[Hz]) \\ 1 (low[Hz] \leq f \leq high[Hz]) \end{cases}$$

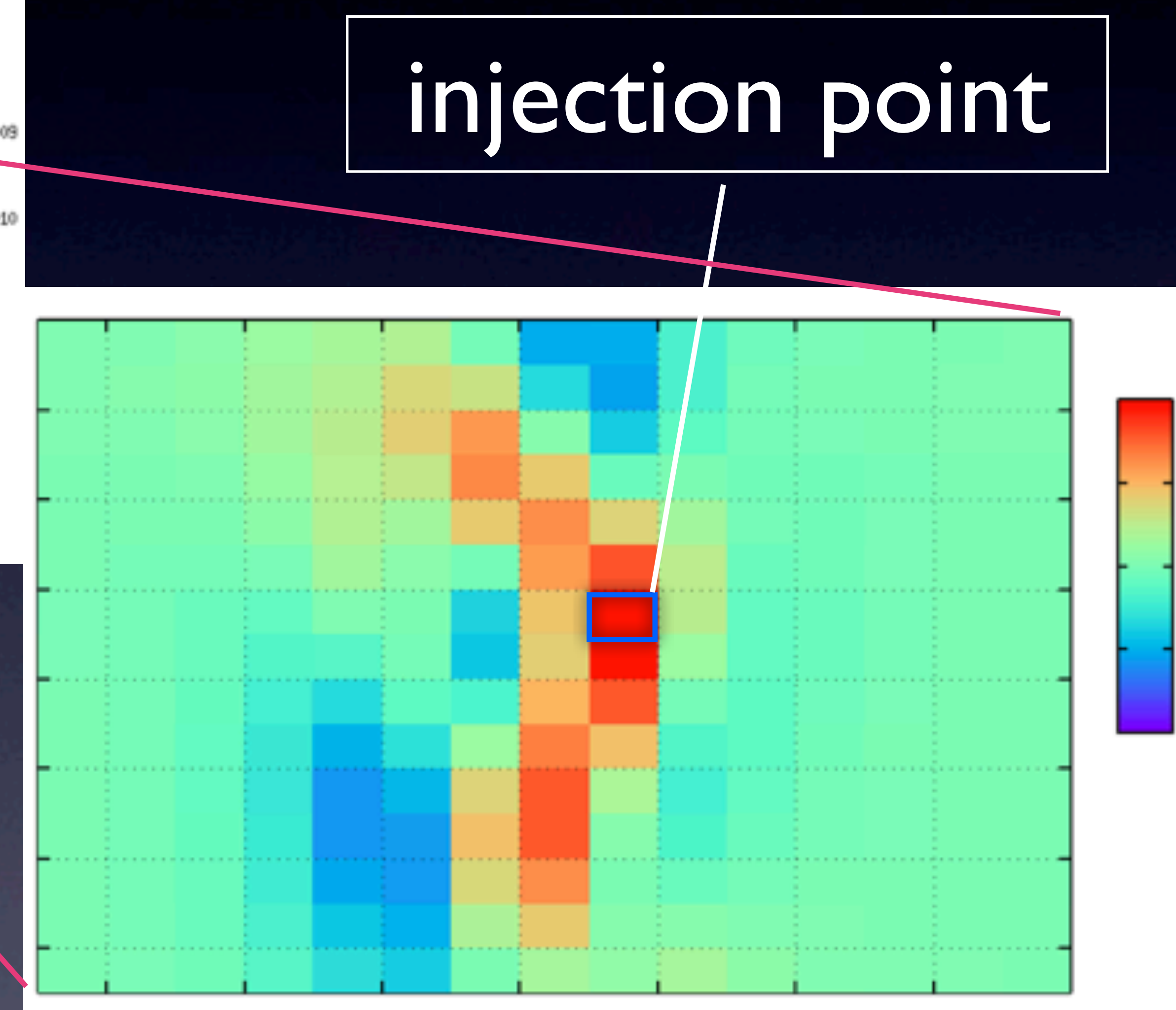
$$\lambda = \frac{1}{\langle \Gamma^2 \rangle^{\frac{1}{2}}}$$

normalization

$$\Gamma = F_1^+ F_2^+ + F_1^\times F_2^\times$$

Qualitative behavior of GW Radiometry





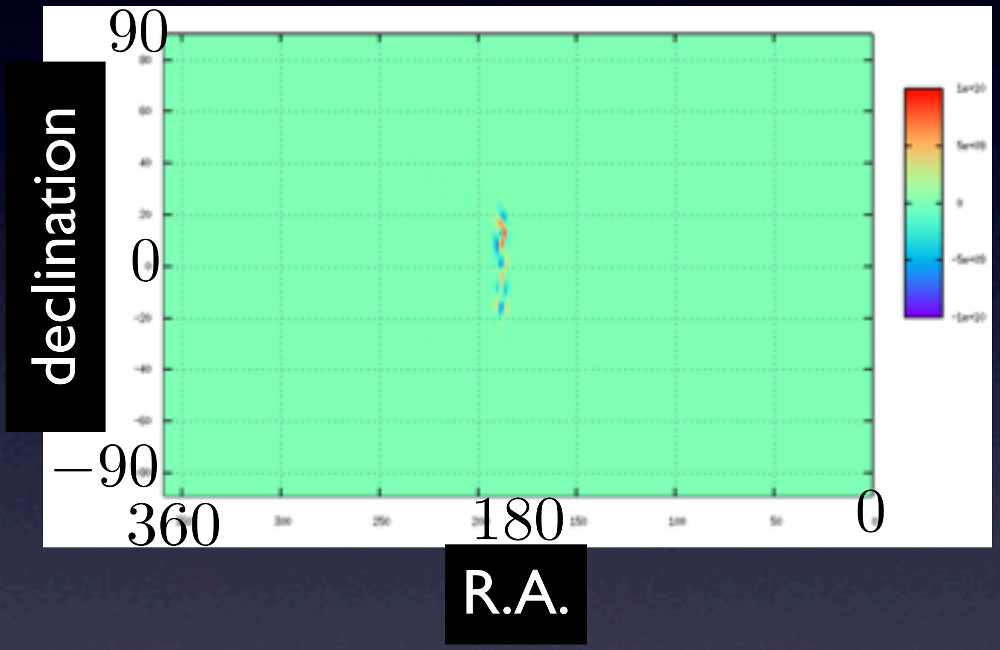
$$\rho(\hat{\Omega}) = \frac{\mu(\hat{\Omega})}{\sigma(\hat{\Omega})}$$

S/N

$$\mu = \langle \Delta S \rangle \, , \sigma^2 = \left\langle [\Delta S - \mu]^2 \right\rangle$$

The problem of GW Radiometry simulation

- For make the map, have to calculate the all direction of the sky in one time



- To detect the stochastic signal, necessary to integrate for a long time (for years)

Much calculation time

Speed up !!

GPGPU

GPGPU＝General Purpose computing on Graphics Processing Units

GPU has much more cores than CPU. (Cores are the head of CPU and GPU.)

GPGPU's strong benefit

- With concurrency, GPGPU allows us to calculate much faster.

example

CPU	GPU
Xeon 5650	Tesla C2075
2.67GHz	1.15GHz
6Cores	448Cores

Have many cores!

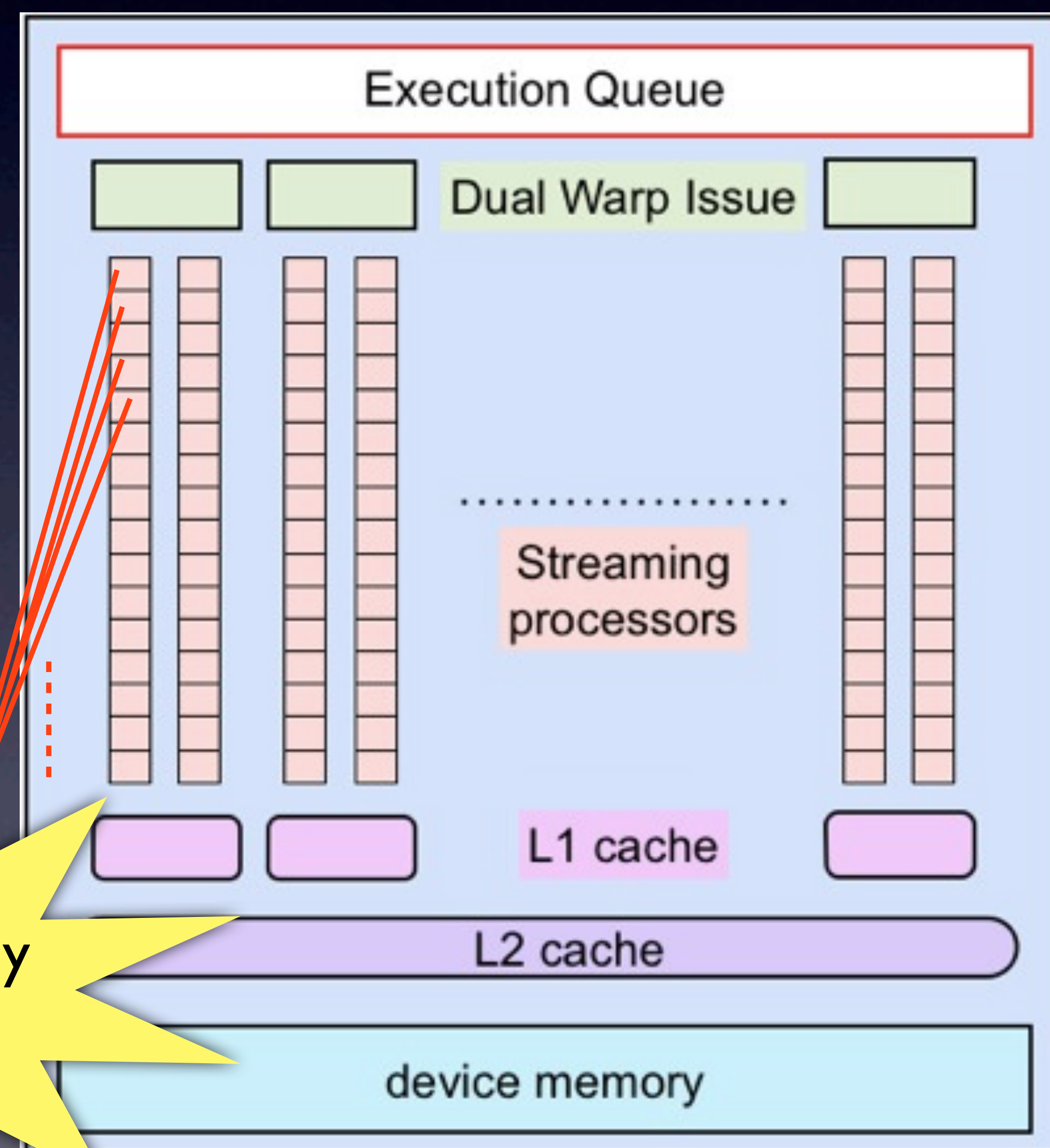
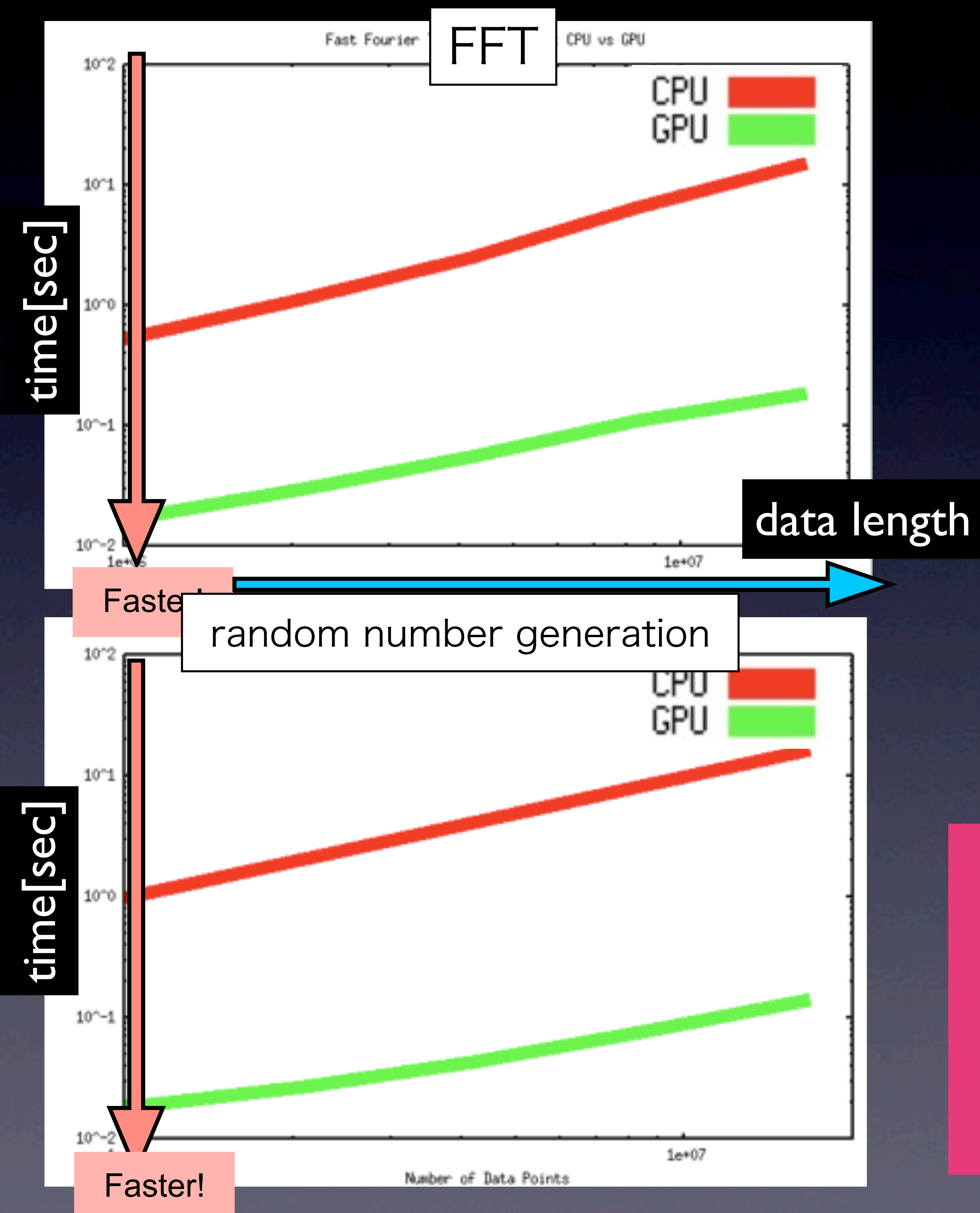


image of GPU

Fast processing with GPGPU



The calculation time of GW Radiometry is dominated by FFT(Fast Fourier Transform) and the random number generation. (Almost 90% of the calculation time.)

FFT, 80 times faster

random number generation, 115 times faster

Achievement

With GPGPU, almost 100 times faster than with CPU in GW Radiometry simulation.

Numerical simulations

- GW from Virgo cluster hotspots
- Source decompose filter

GW from Virgo cluster hotspots

There are some previous works by other people.

S. Dhurandhar et al. , Phys. Rev. D 84, 083007(2011)

Sanjit Mitra et al., Phys. Rev. D 77, 042002(2008)

Yuta Okada, Master thesis(2012)

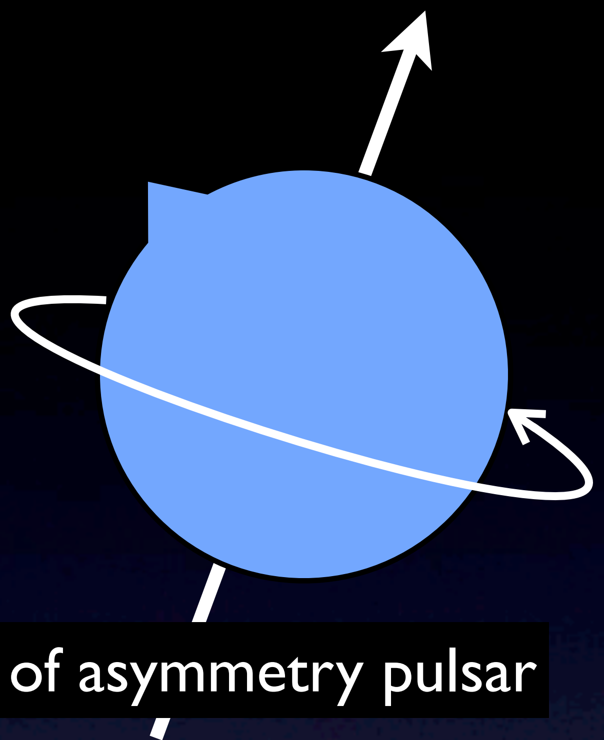


Image of asymmetry pulsar

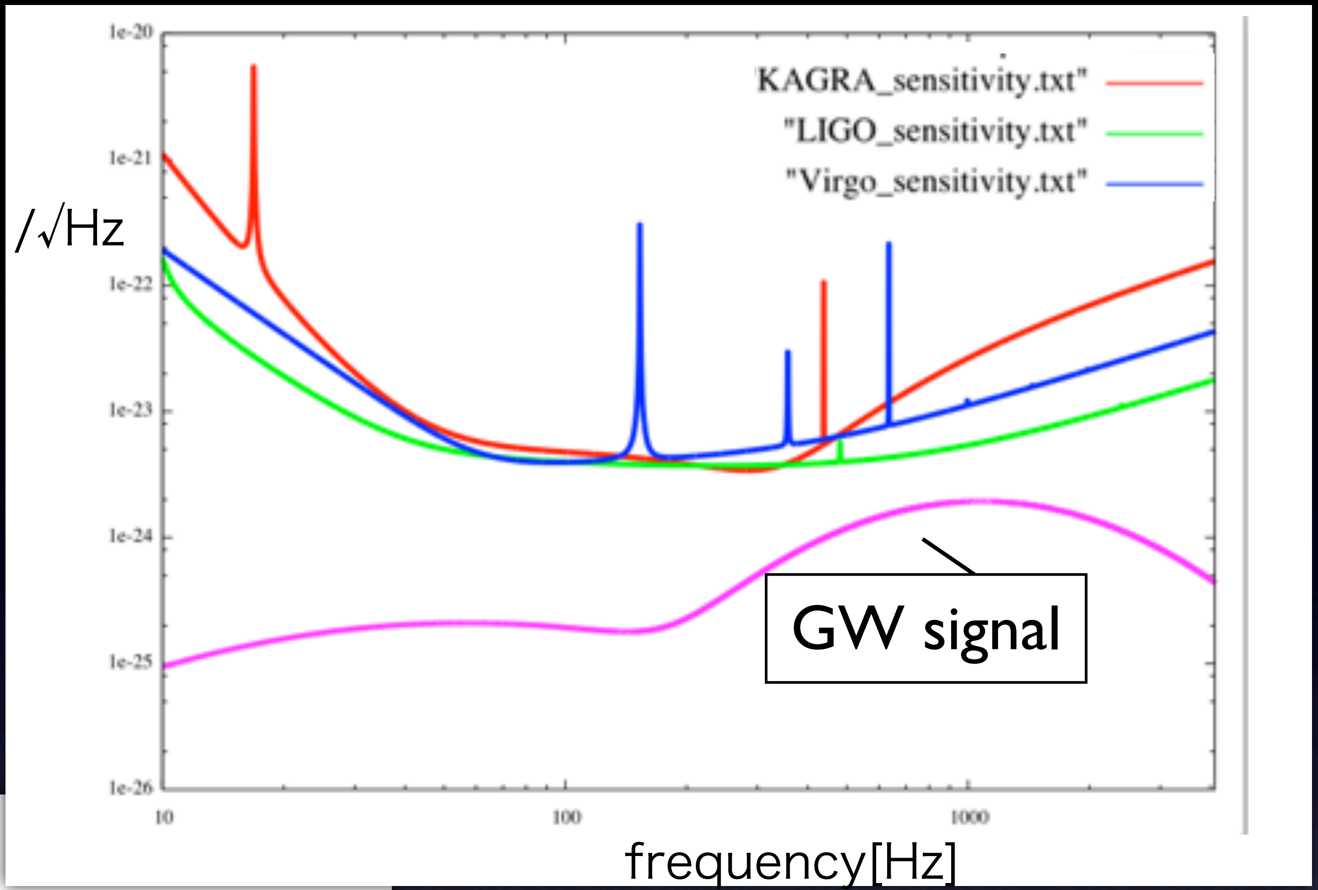
Simplify reasons to choosing

1. Asymmetry pulsars emit GW
2. Virgo cluster has a lot of pulsars(~about 10e11 ?) in particular direction of the sky
3. The merger of those GW will be a good source of stochastic GW

GW from Virgo cluster hotspots

Radiometry filter

$$\tilde{Q}(f,\hat{\Omega};t) = \lambda \frac{\gamma^*(f,\hat{\Omega})H(f)}{P_1(f)P_2(f)}$$



Window function

$$H(f) = \begin{cases} 0(f < 200, f > 1500[Hz]) \\ 1(200 \leq f \leq 1500[Hz]) \end{cases}$$

GW signal

$$\begin{aligned} Signal(f) &= \langle h^2 \rangle N(f) \\ &= \left[7.05 \times 10^{-34} \left(\frac{\epsilon}{10^{-5}} \right) \left(\frac{I}{1.1 \times 10^{45} \mathrm{gcm}^2} \right) \right]^2 \\ &\quad \times \langle \alpha^2 \rangle f^4 N(f) \ [\mathrm{Hz}^{-1}] \end{aligned}$$

I : Inertial moment, here $I = I_{zz}$ $\langle \alpha^2 \rangle = 0.4$ $N(f)$: number of pulsars

$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$ f : GW's frequency

R.A. : 12h 27m 13s
declination : +12° 4' 59" : sky position

S. Mitra et al. Phys. Rev. D 77, 042002(2008)
S. Dhurandhar et al. Phys. Rev. D 84, 083007(2011)

GW from Virgo cluster hotspots

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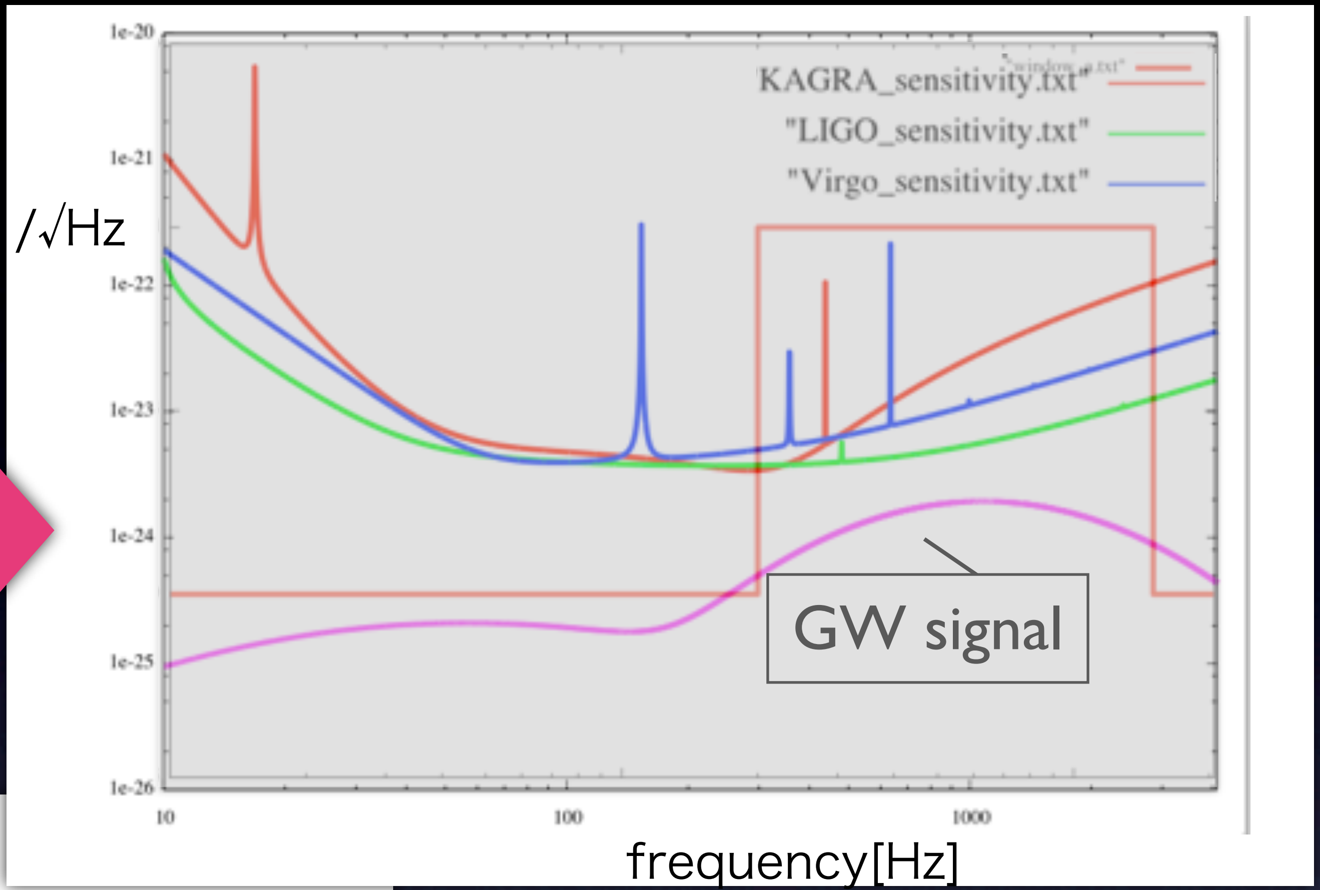

GW signal

$$Signal(f) = \langle h^2 \rangle N(f)$$
$$= \left[7.05 \times 10^{-34} \left(\frac{\epsilon}{10^{-5}}\right) \left(\frac{I}{1.1 \times 10^{45}\mathrm{gcm}^2}\right)\right]^2$$
$$\times \langle \alpha^2 \rangle f^4 N(f) \ [\mathrm{Hz}^{-1}]$$

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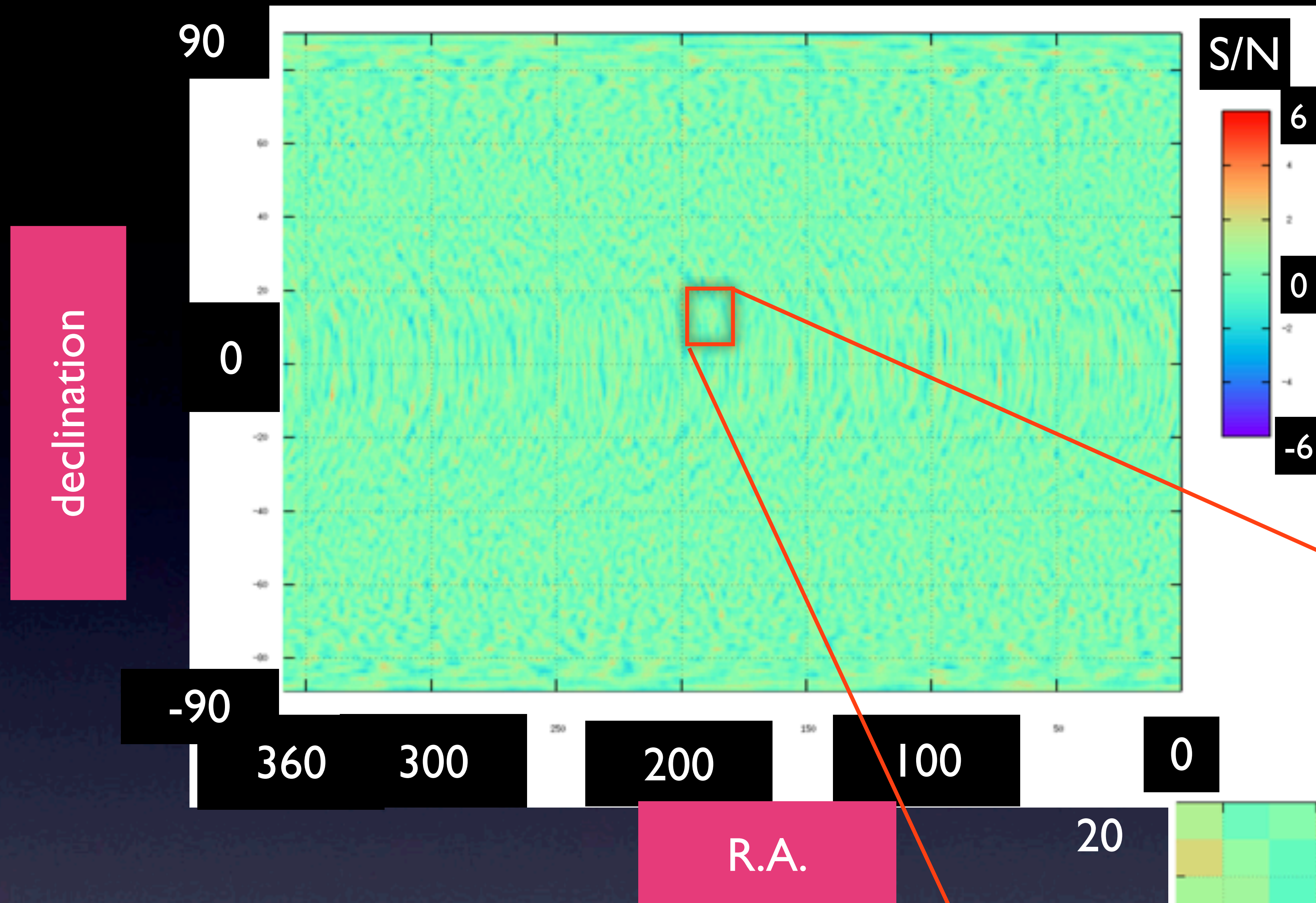
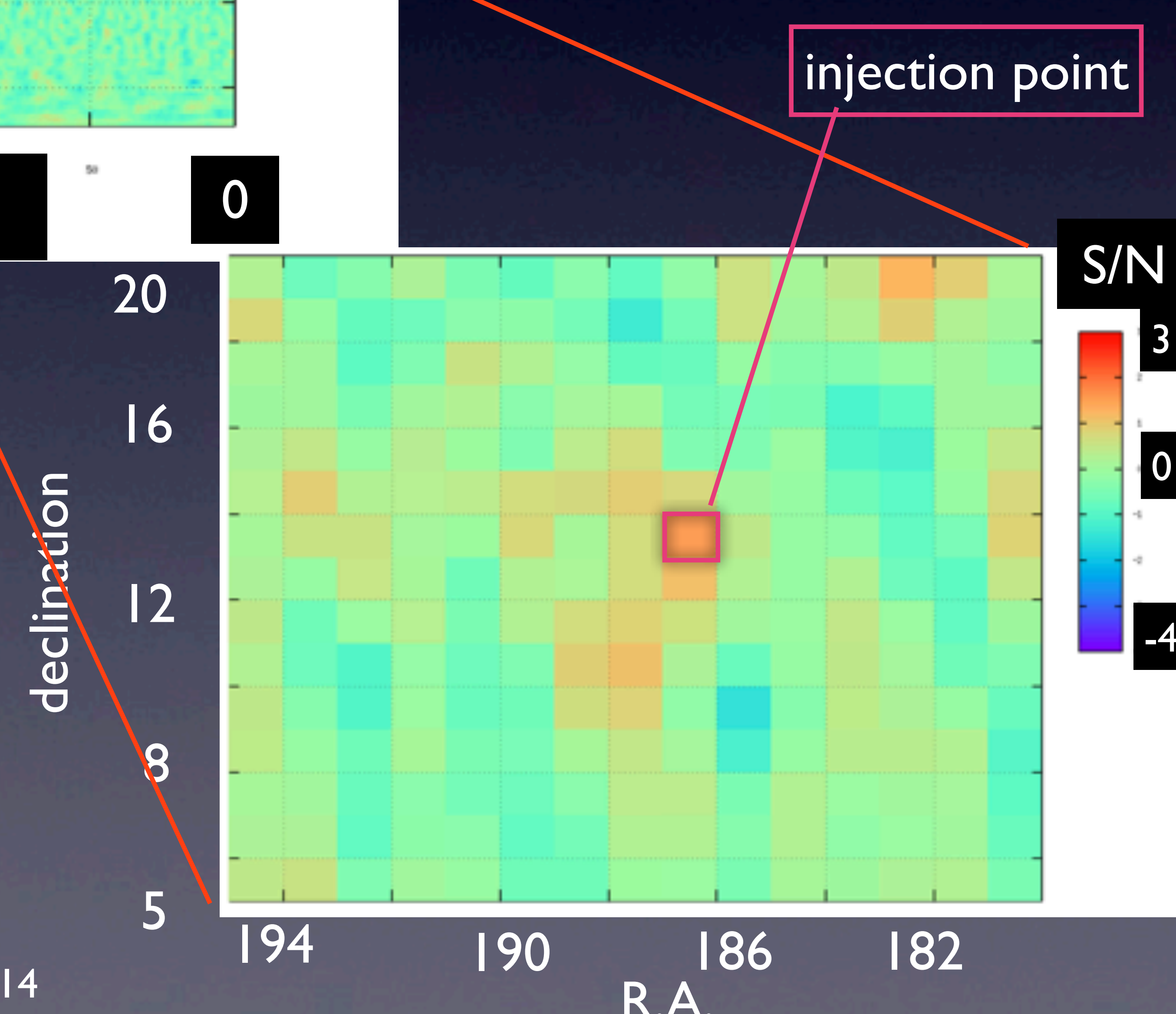
KAGRA-LIGO Livingstone combination
1 year simulation's output

x axis is Right Ascension
y axis is declination
color is signal to noise ratio

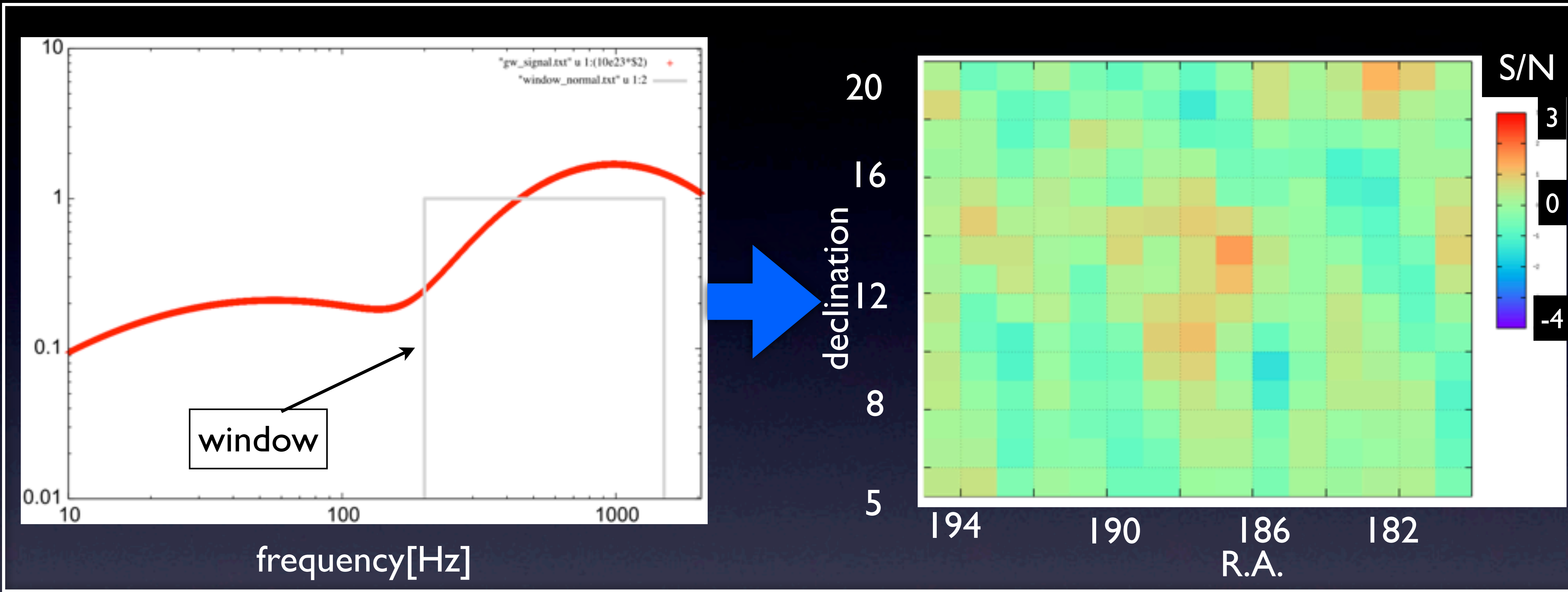




Signal to noise ratio of the injection point is 2.3 . It is consistent to the previous study.(Yuta Okada, Master thesis(2012))

And the calculation time becomes much shorter.(about 5 days > about 1 hour and 5 minutes)

Source decompose filter



Typical GW Radiometry map

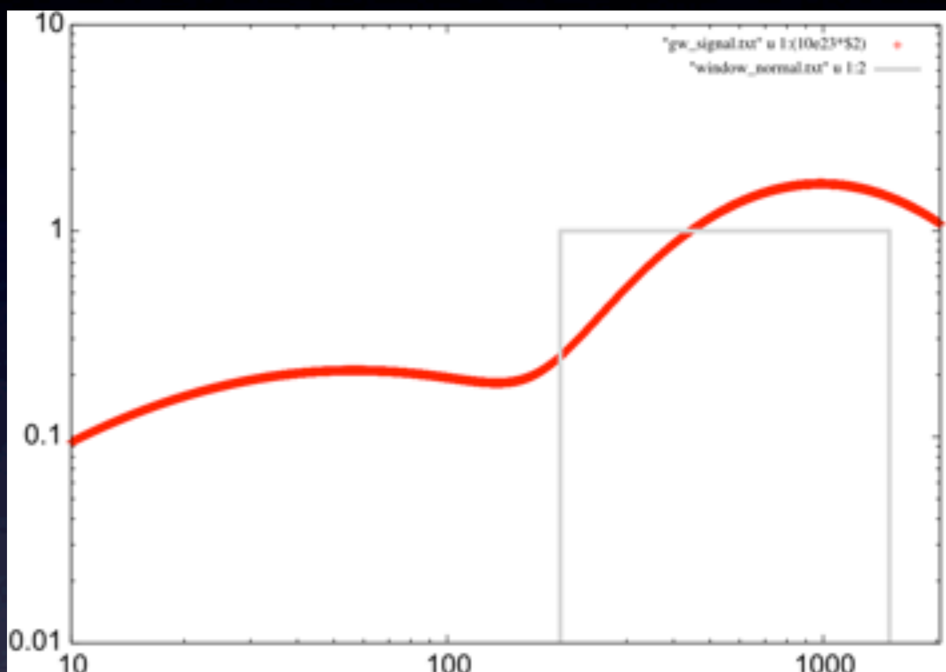
The left figure is typical type of GW Radiometry window function.
It has wide frequency band.(200 ~ 1500[Hz])

The right figure is the signal to noise ratio map around the GW injection point.

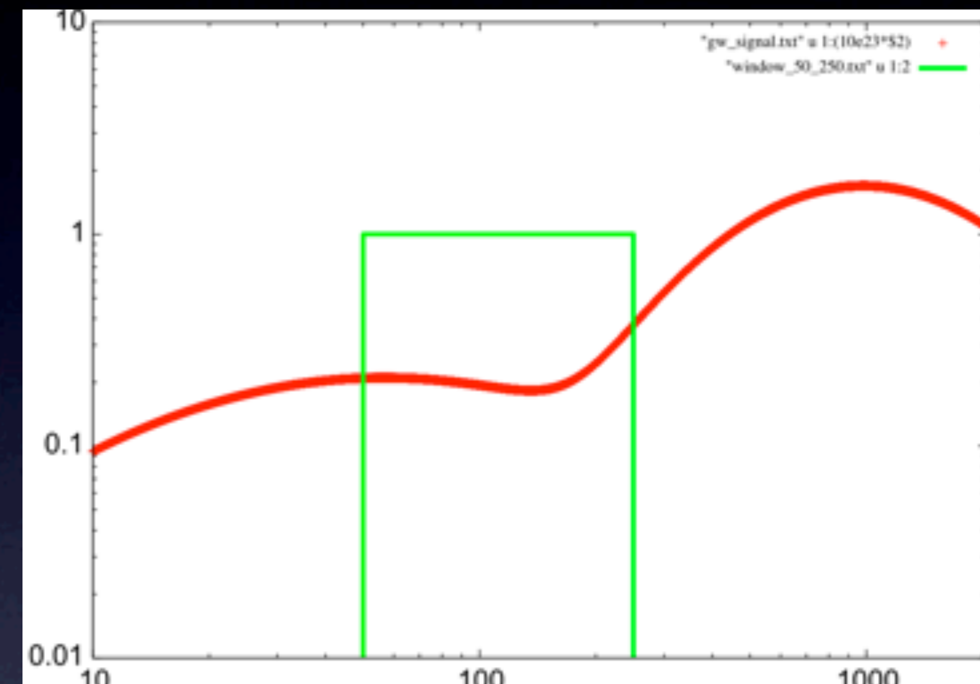
Source decompose filter

Here we use the some kinds of window function.

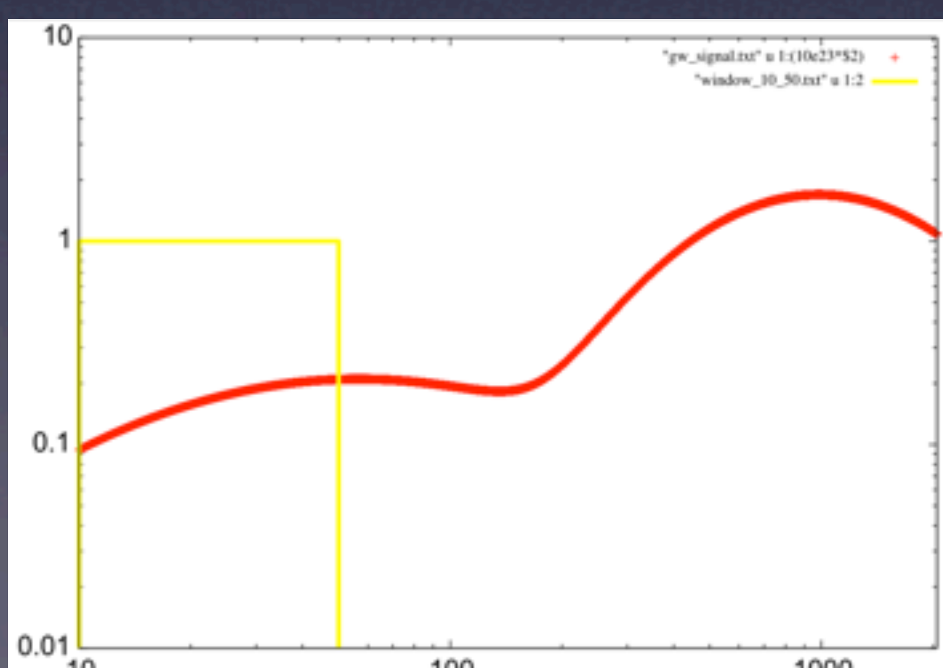
- Normal window 200~1500[Hz]



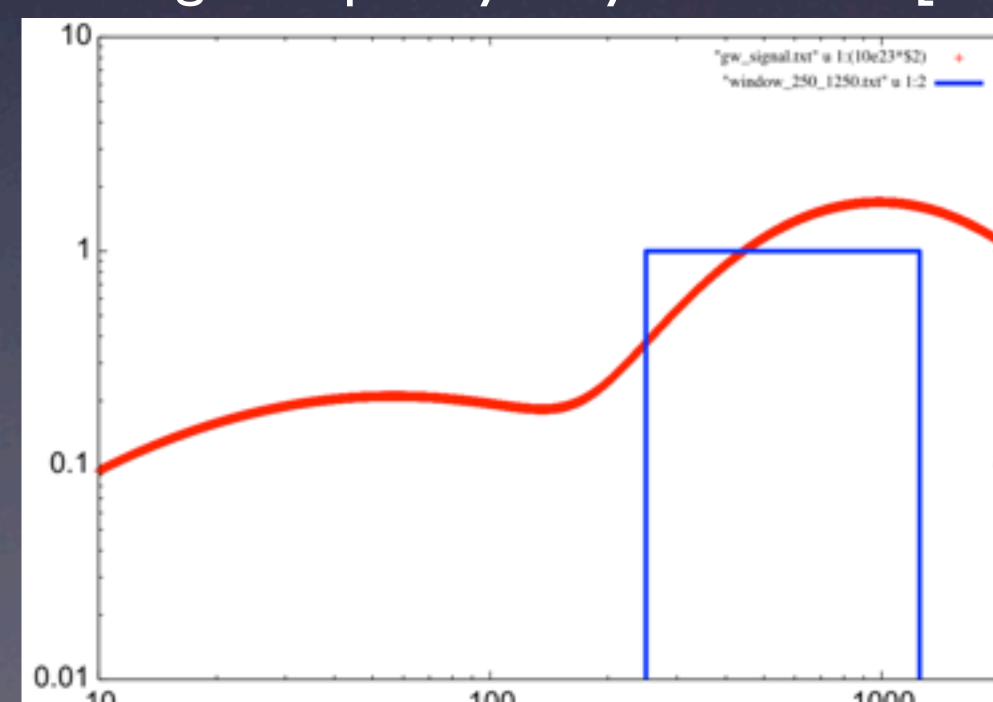
- Middle frequency only 50~250[Hz]



- Low frequency only 10~50[Hz]

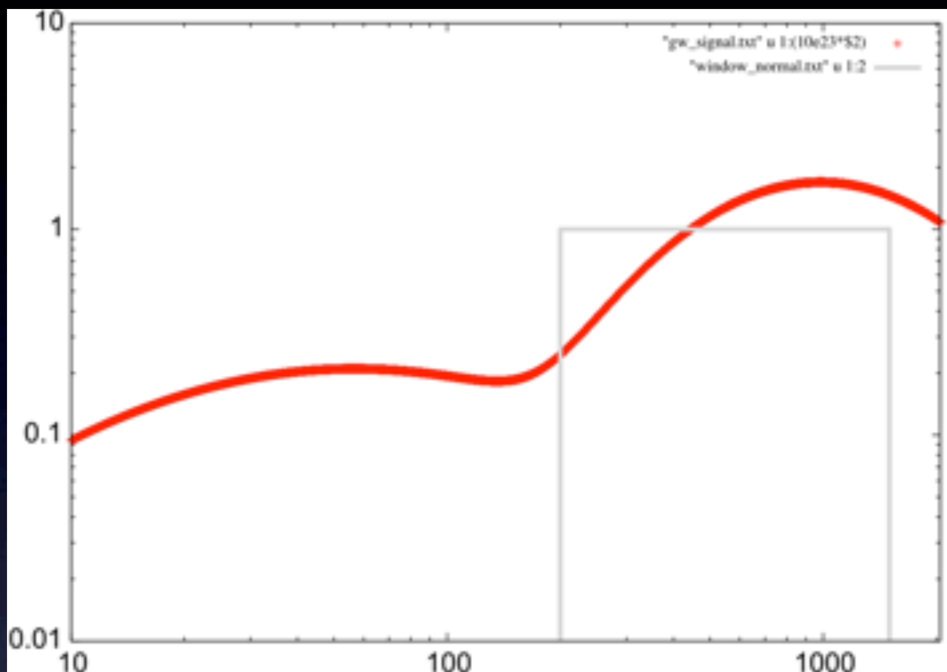


- High frequency only 250~1250[Hz]

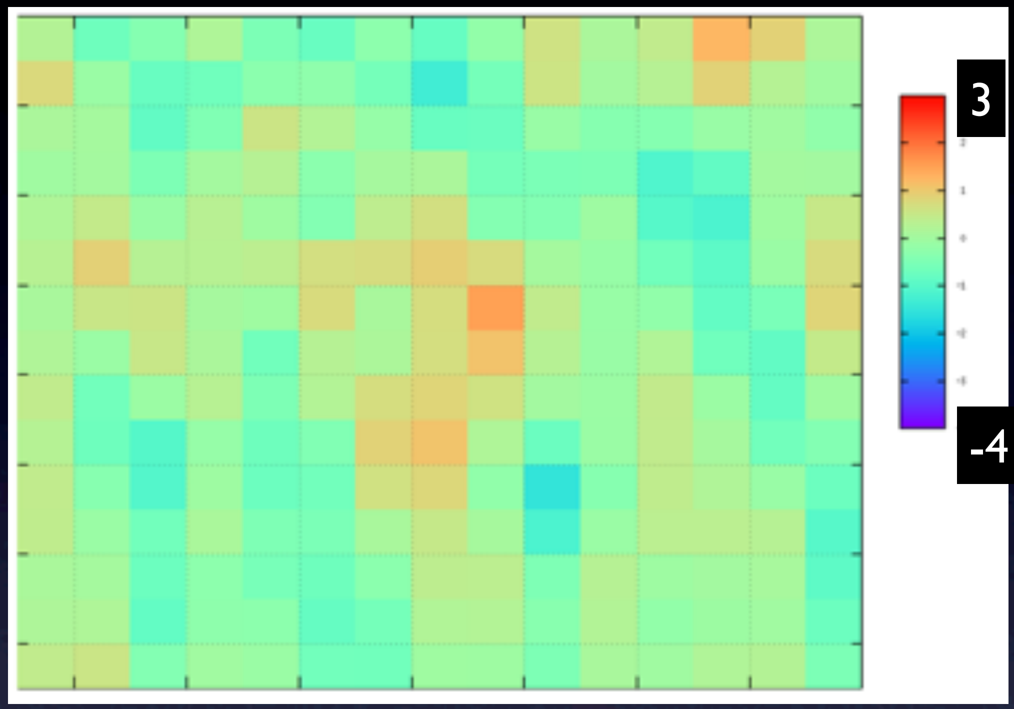


Source decompose filter

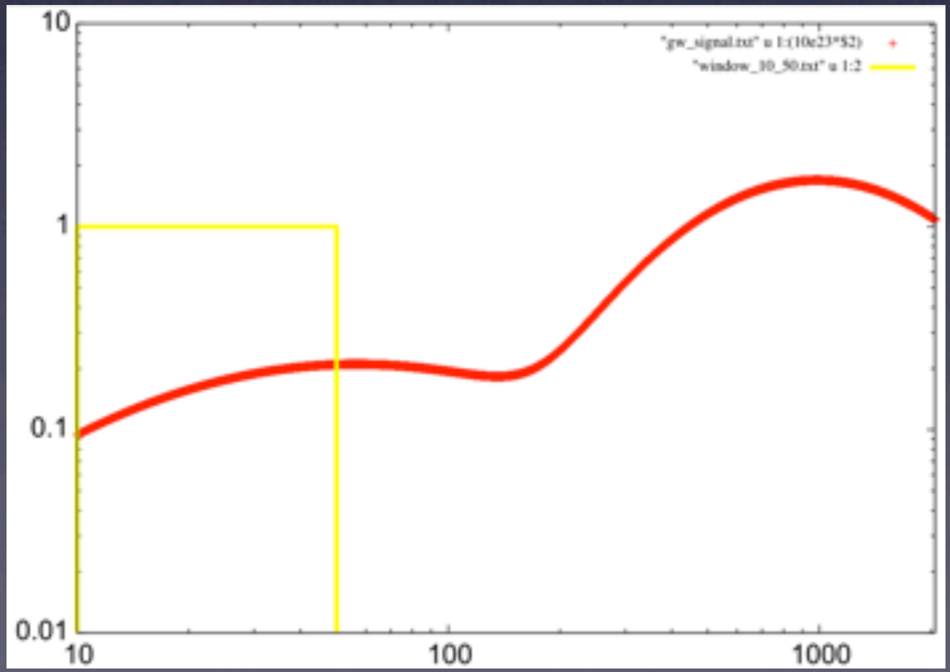
Normal



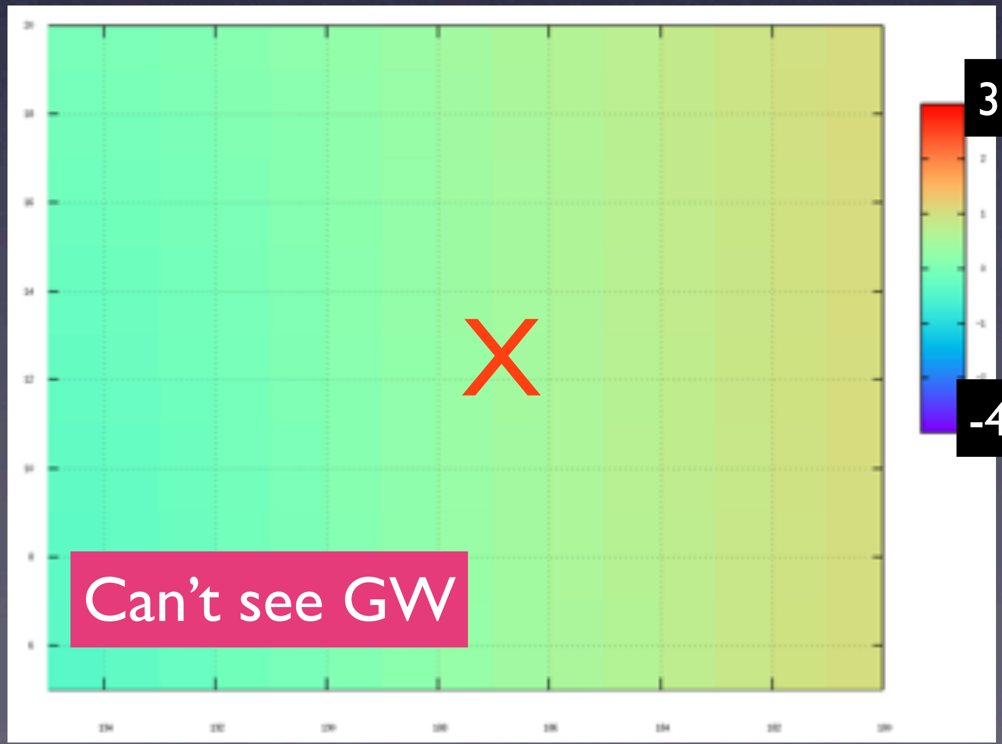
Zoom map around the GW injection point



Low

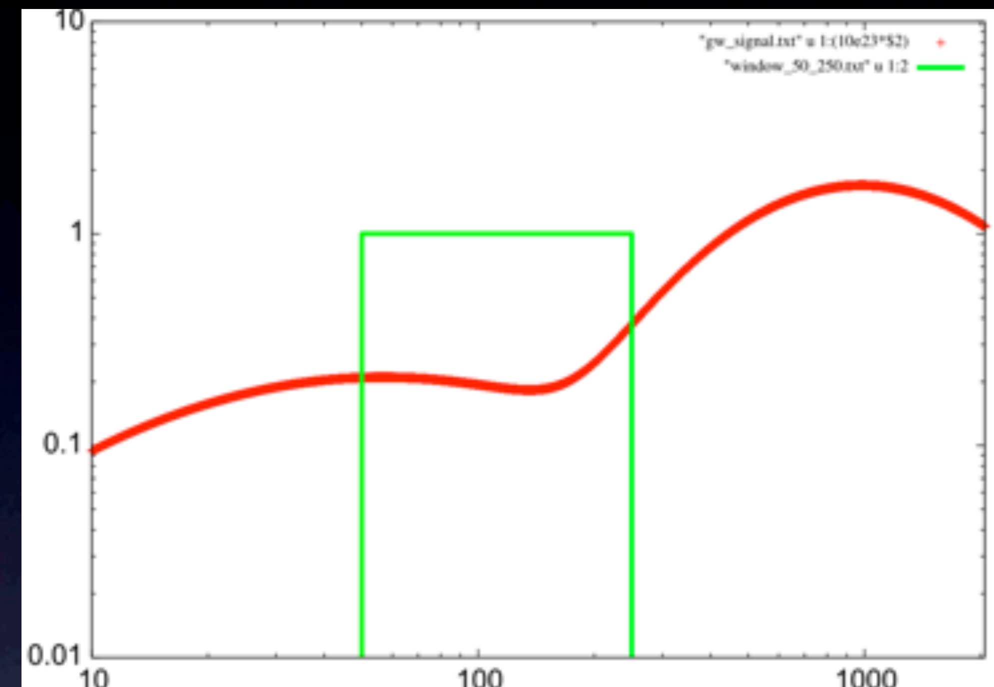


Zoom map around the GW injection point

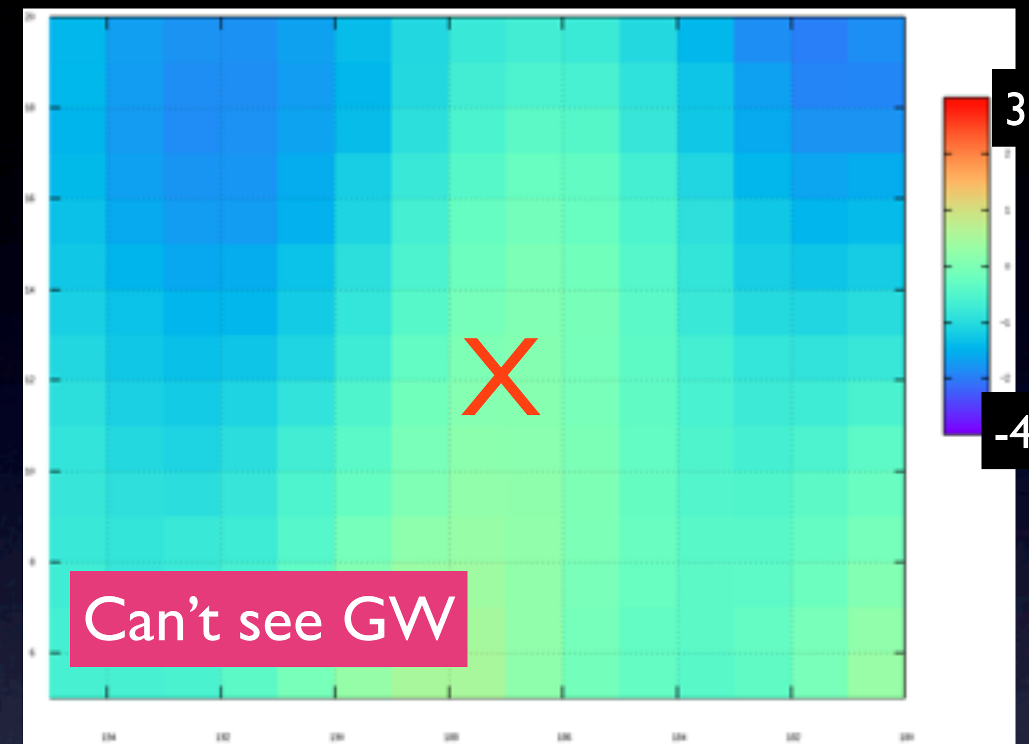


Source decompose filter

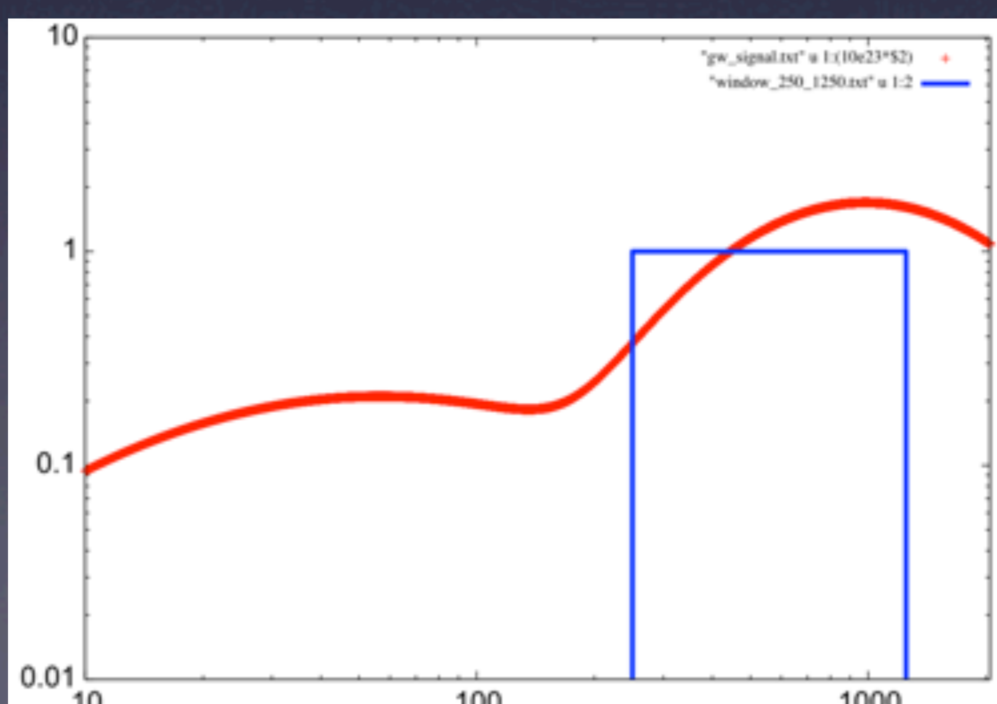
Middle



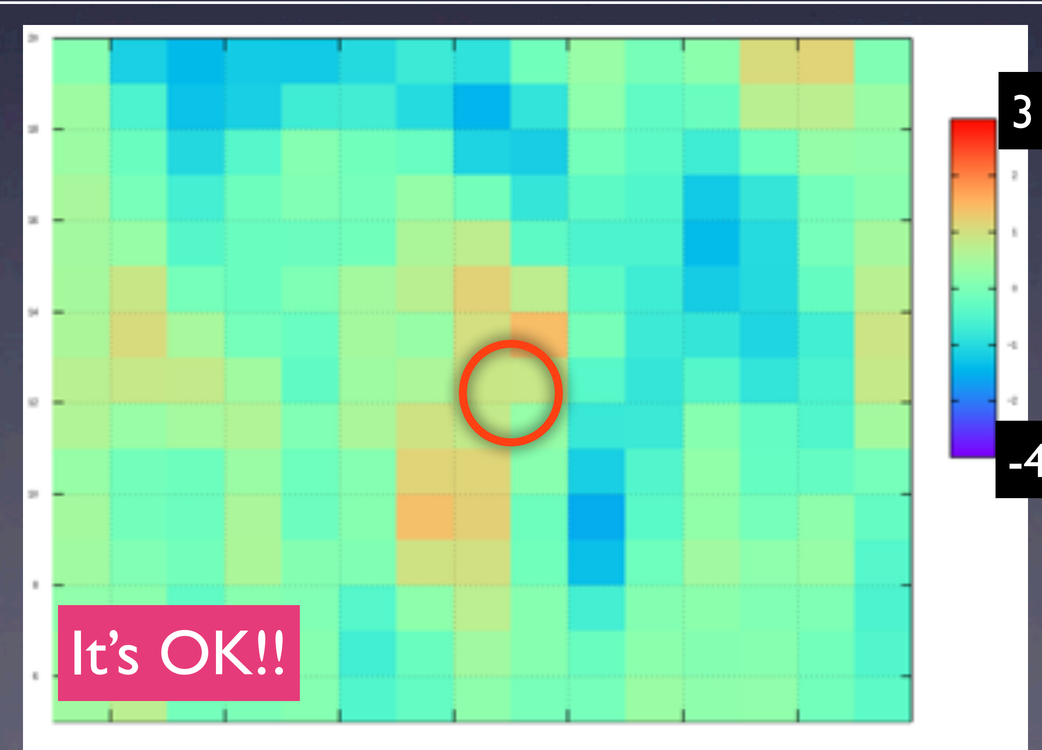
Zoom map around the GW injection point



High

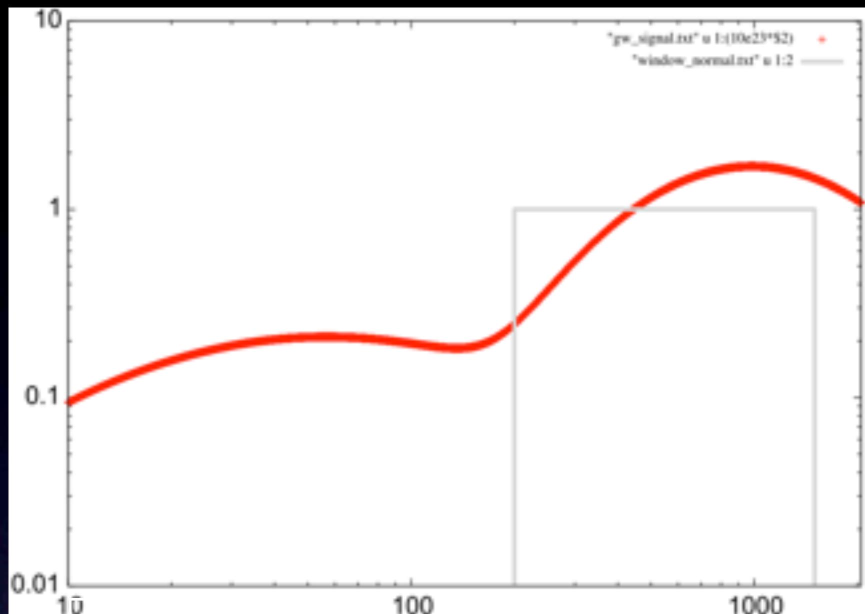


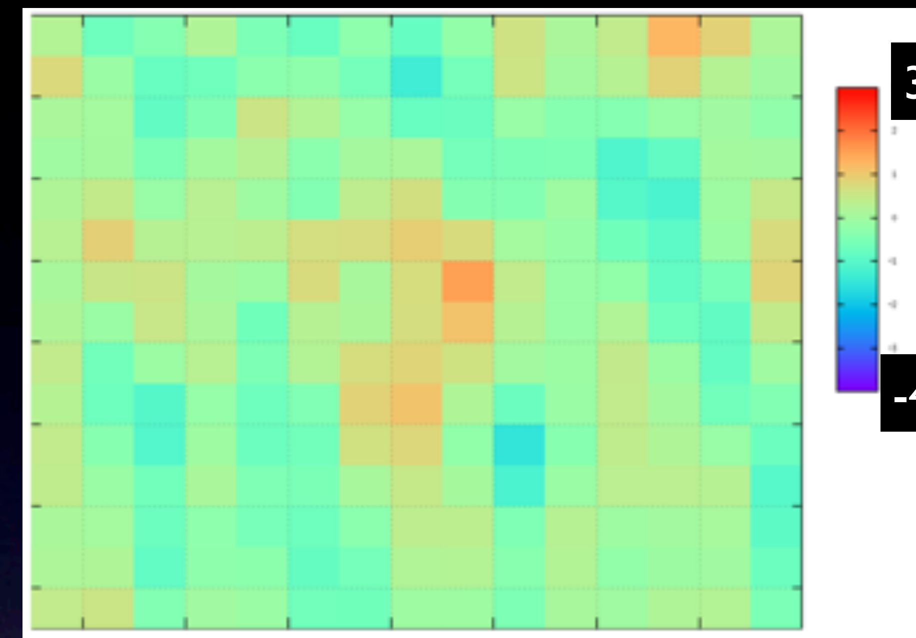
Zoom map around the GW injection point



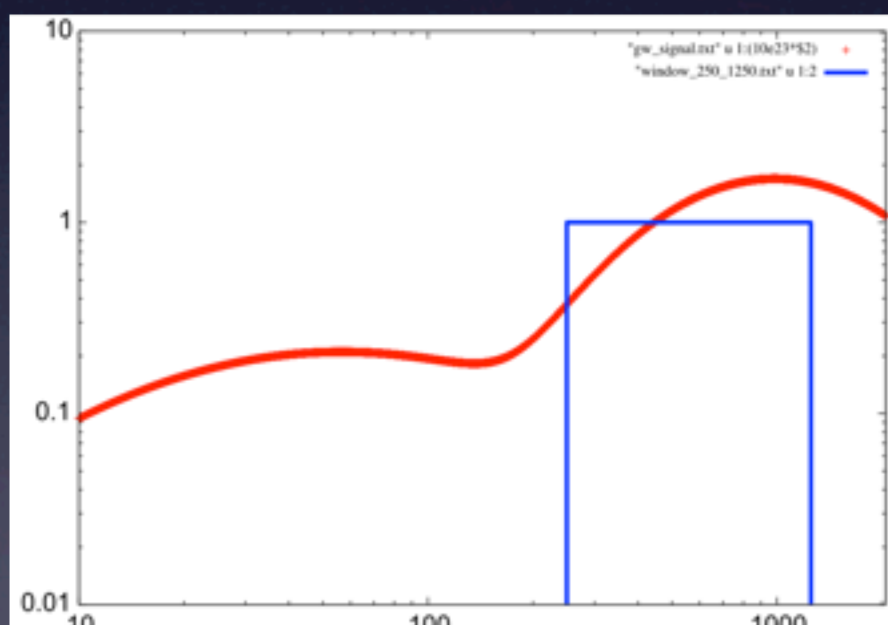
Source decompose filter

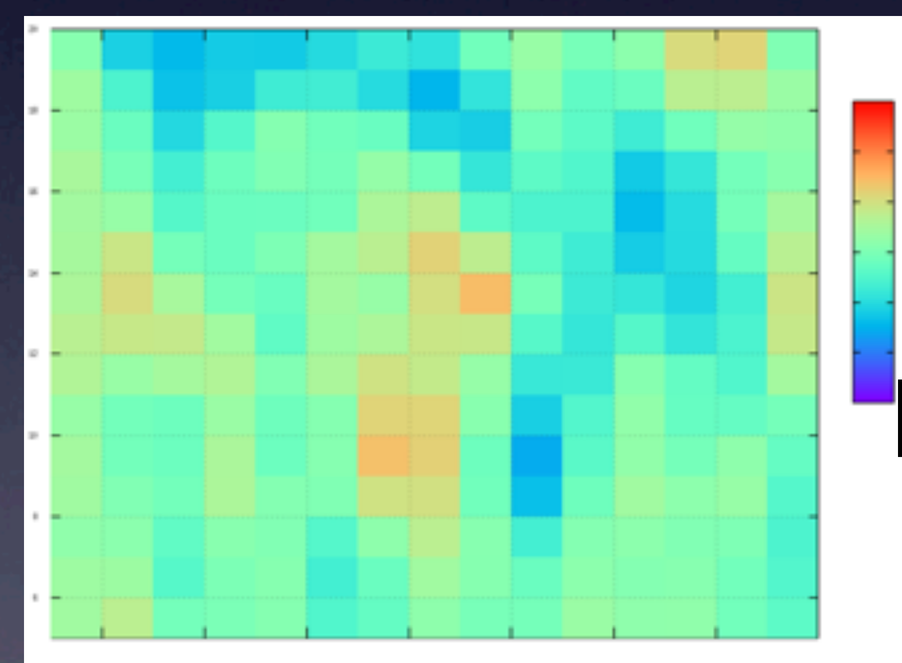
Normal





High





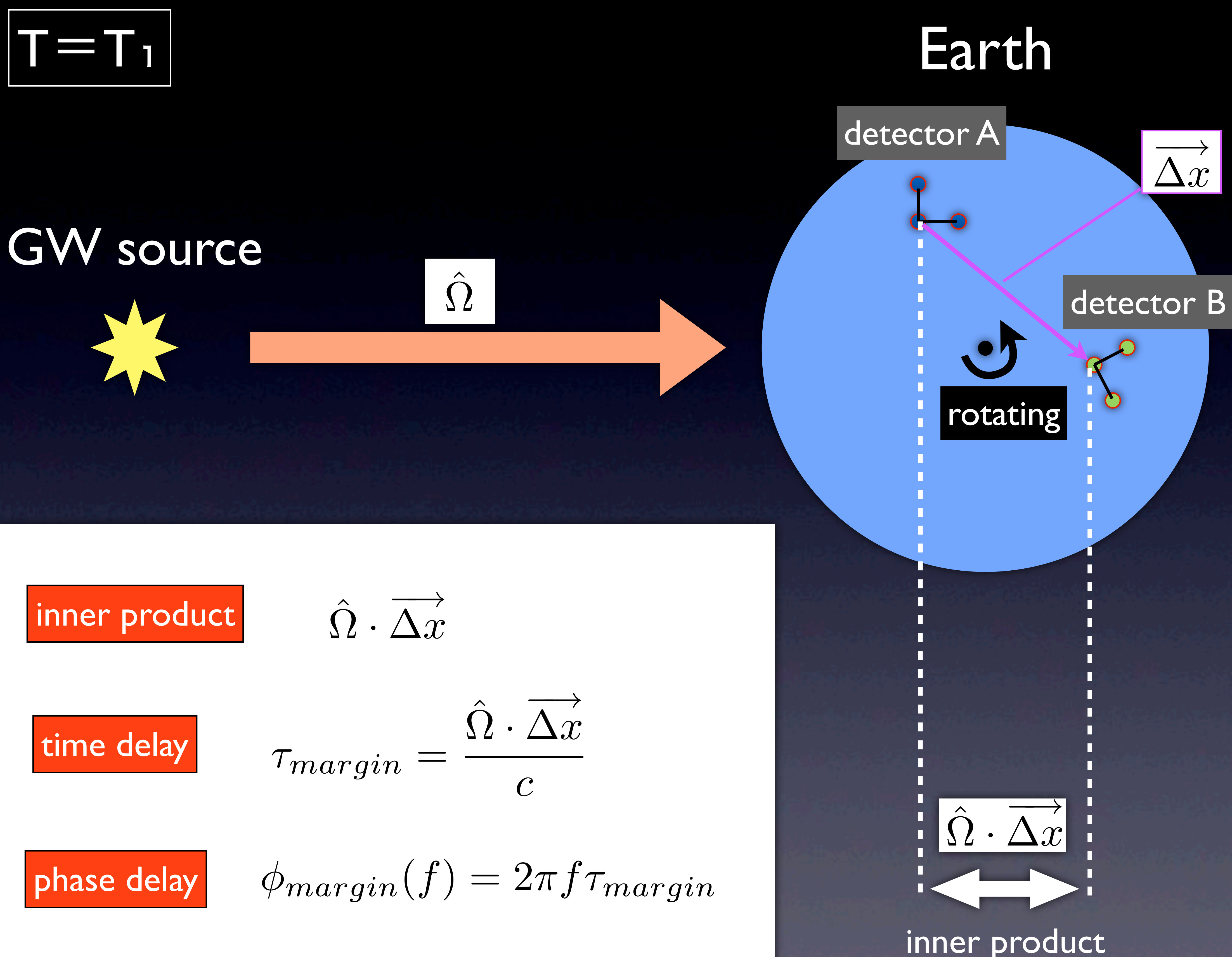
By using such kinds of window function, we can know the signal to noise ratio of this GW is depend on the high frequency.

We can decompose the GW by using well H(f) window function.

Summary and future

- In GW Radiometry simulation : Achieve 100 times faster processing than before.
- Verify the consistence
- Show it is possible to divide the GW by frequency with source decompose filter

- Consider about the spread source
- Simulate the case which GW comes from two or more sources



inner product $\hat{\Omega} \cdot \overrightarrow{\Delta x}$

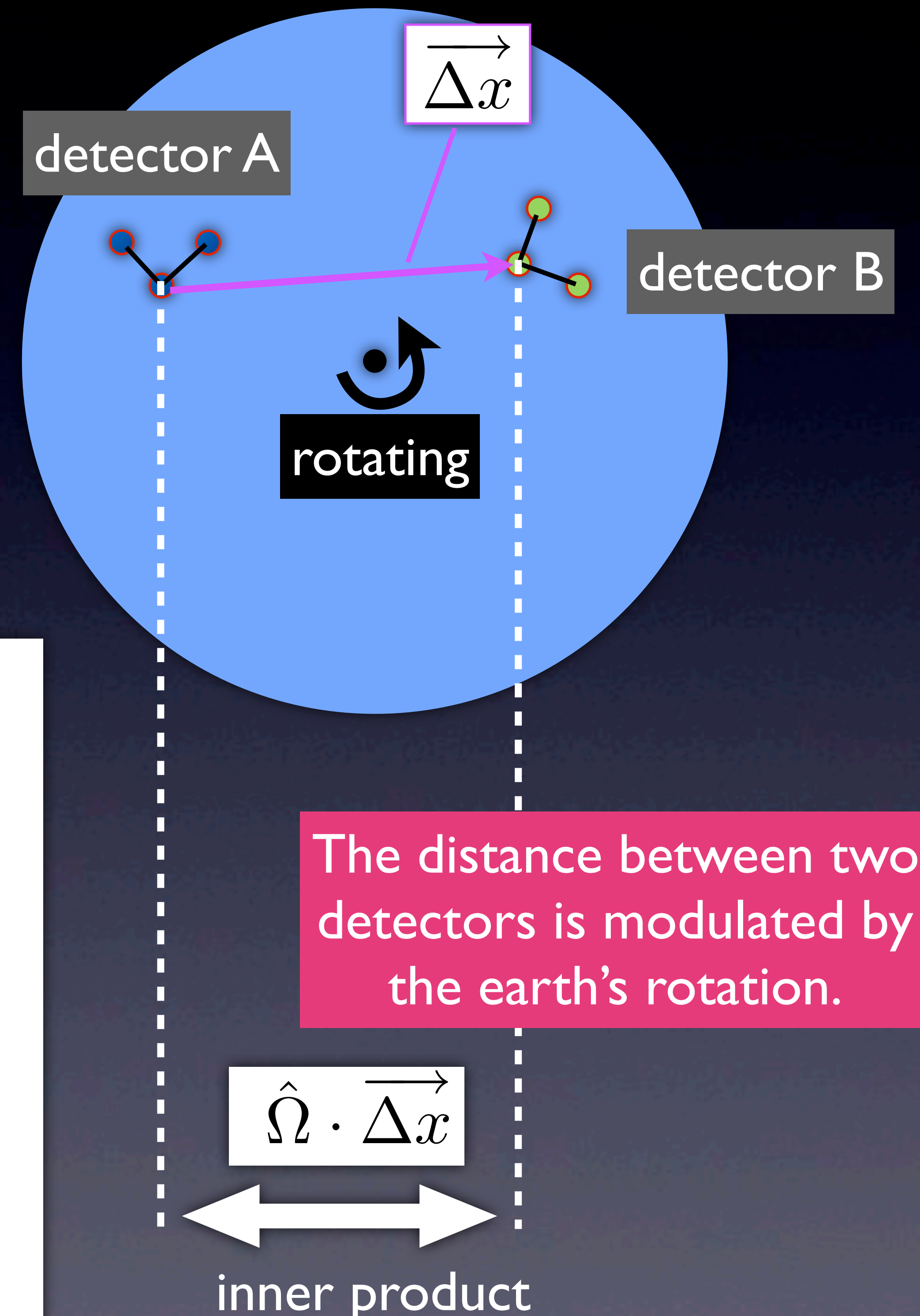
time delay $\tau_{margin} = \frac{\hat{\Omega} \cdot \overrightarrow{\Delta x}}{c}$

phase delay $\phi_{margin}(f) = 2\pi f \tau_{margin}$

T = T2

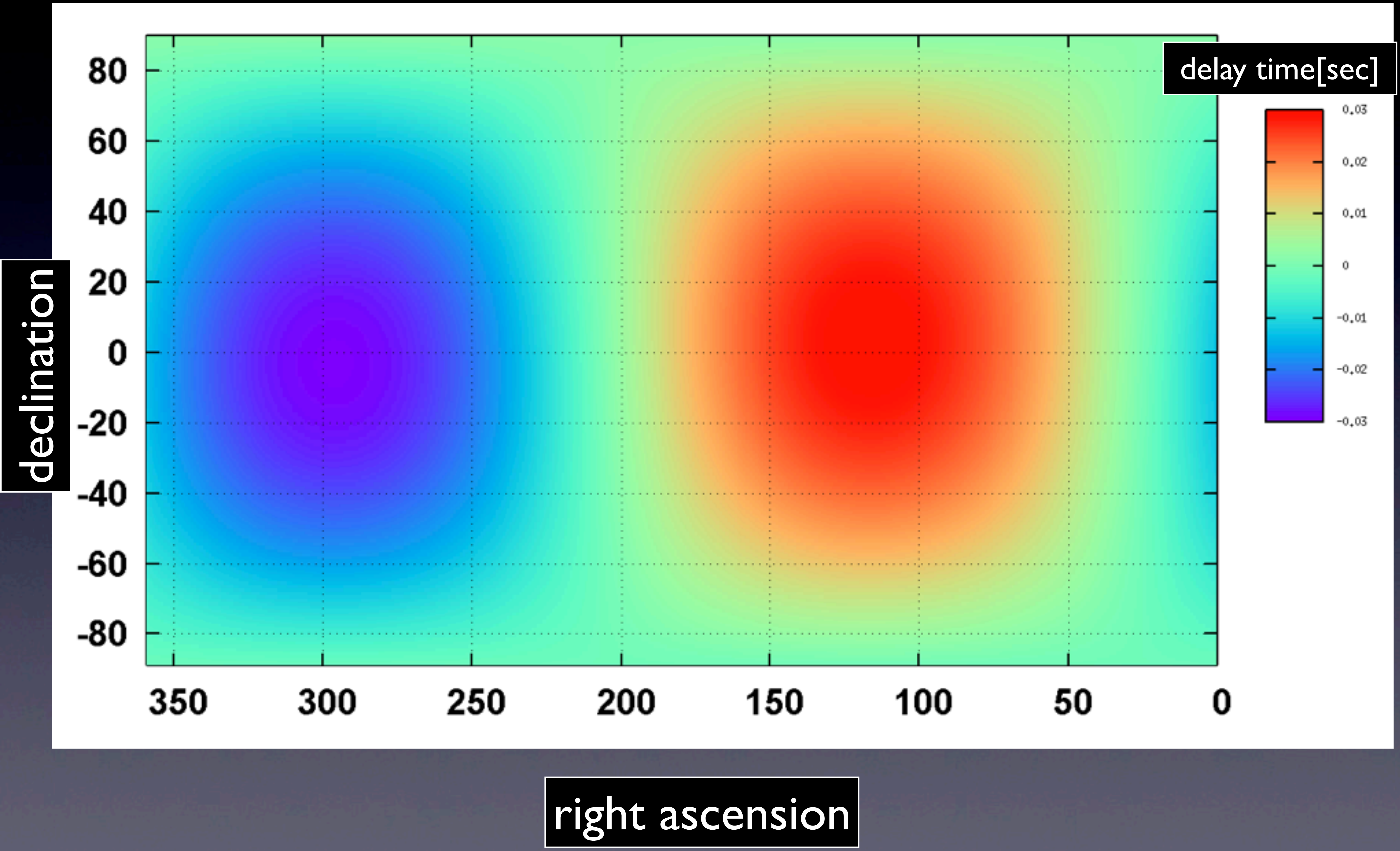
GW source

Earth

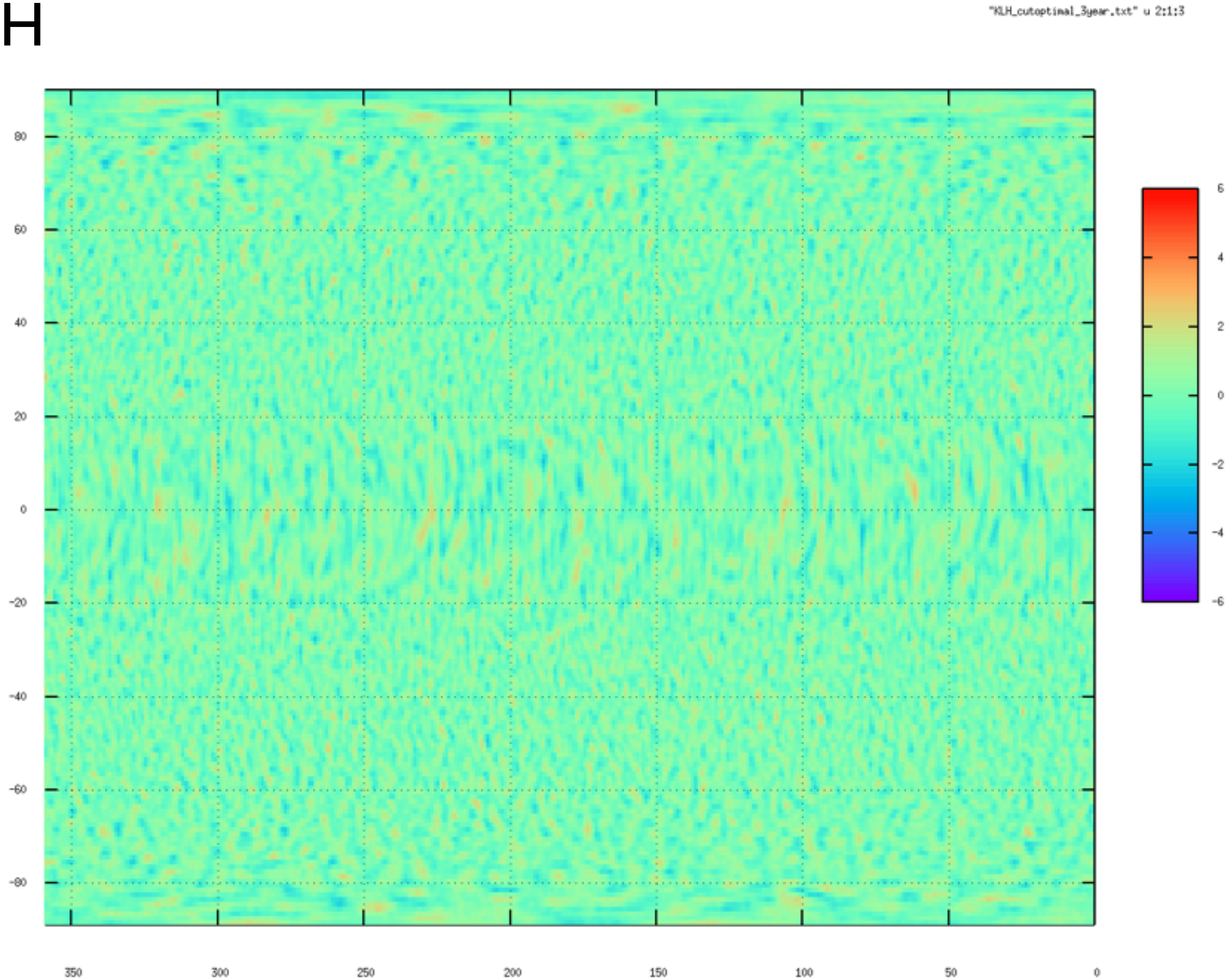


inner product	$\hat{\Omega} \cdot \overrightarrow{\Delta x}$
time delay	$\tau_{margin} = \dfrac{\hat{\Omega} \cdot \overrightarrow{\Delta x}}{c}$
phase delay	$\phi_{margin}(f) = 2\pi f \tau_{margin}$

margin map


delay time[sec]
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
80
60
40
20
0
-20
-40
-60
-80
declination
350
300
250
200
150
100
50
0
right ascension

K-LH


"KLH_cutoptimal_3year.txt" u 2:1:3
80
60
40
20
0
-20
-40
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6
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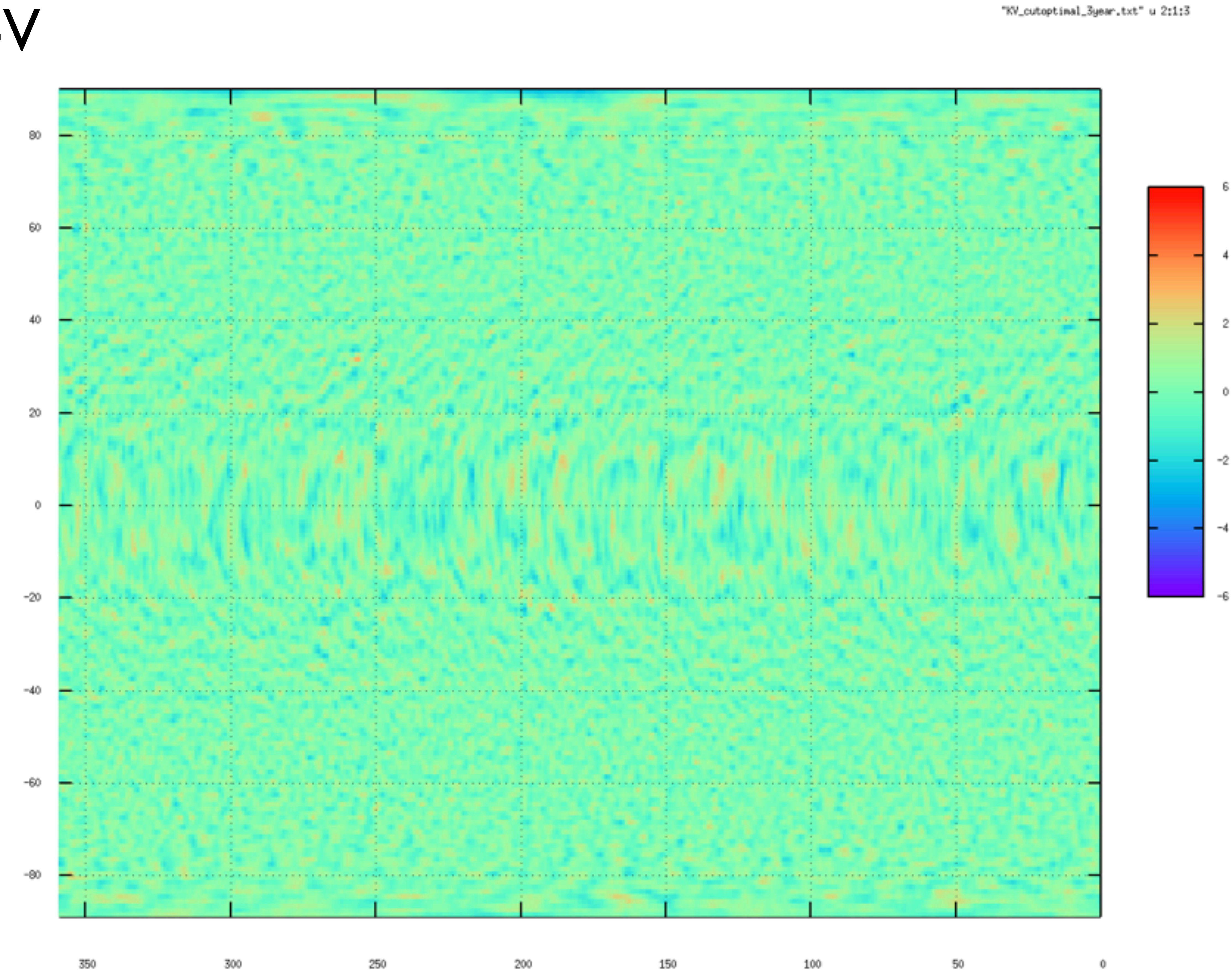
-4
-6

K-V



"KV_cutoptimal_3year.txt" u 2:1:3
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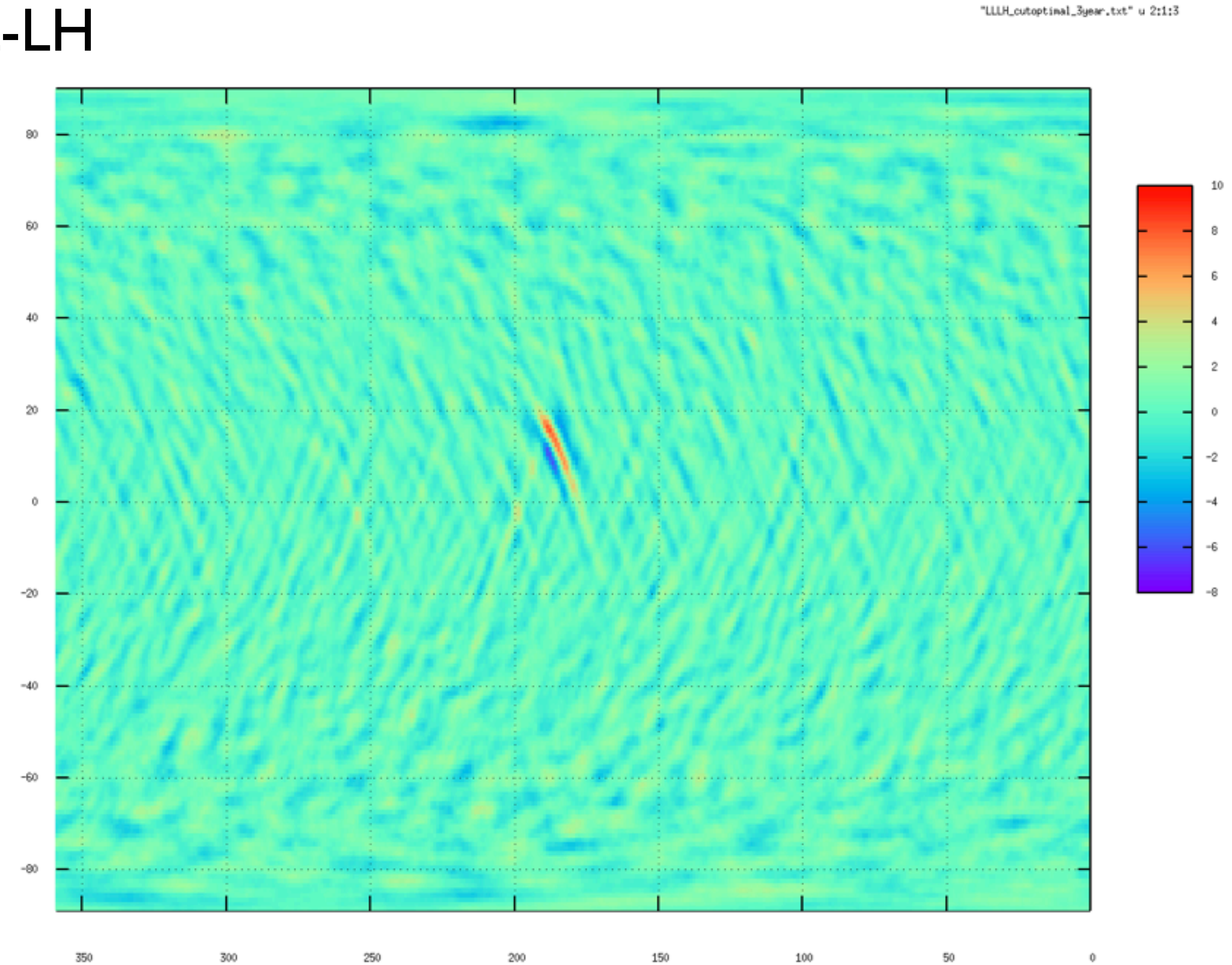
-2
-4
-6

LL-LH



"LLLH_cutoptimal_3year.txt" u 2:1:3
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-4
-6
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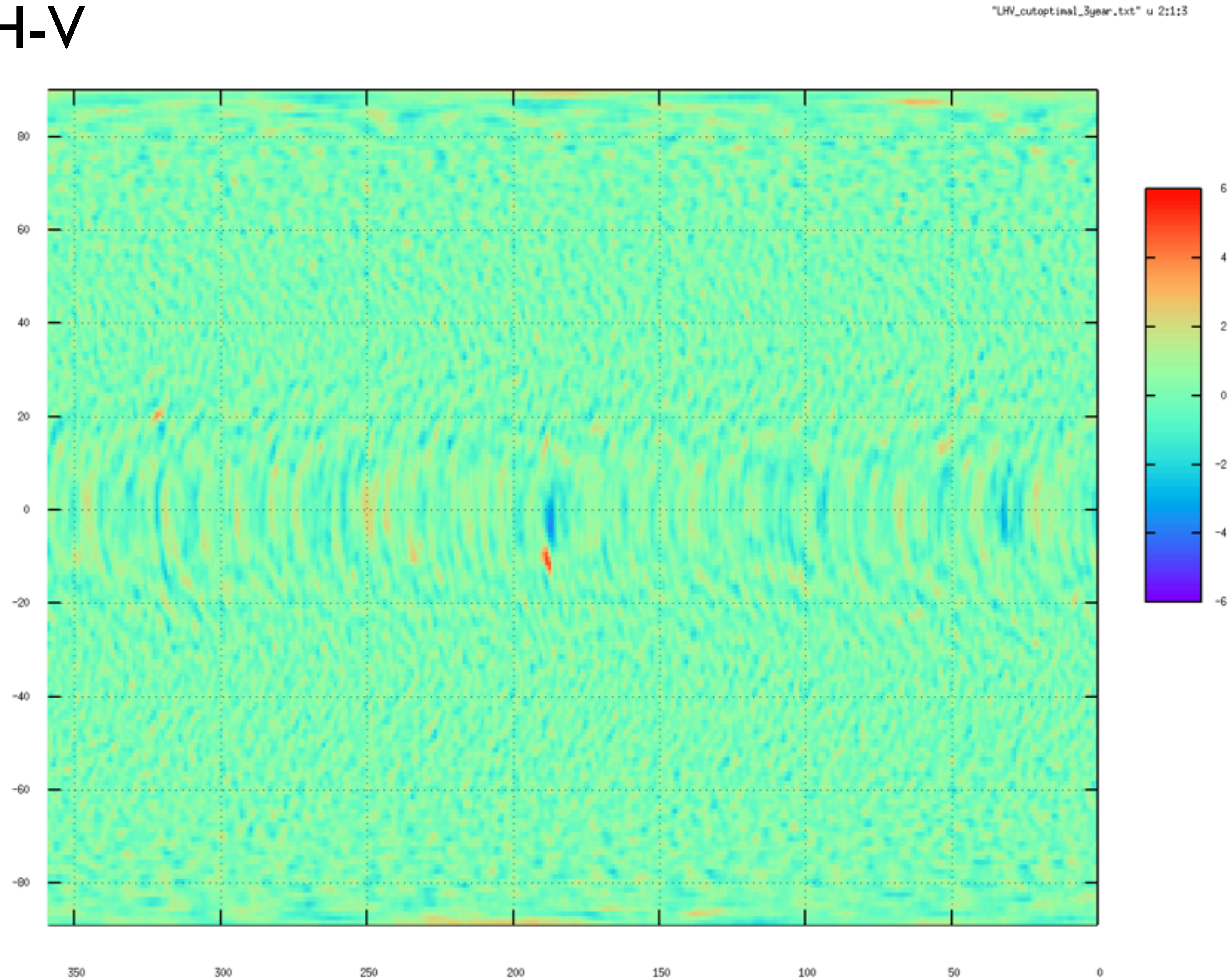
LH-V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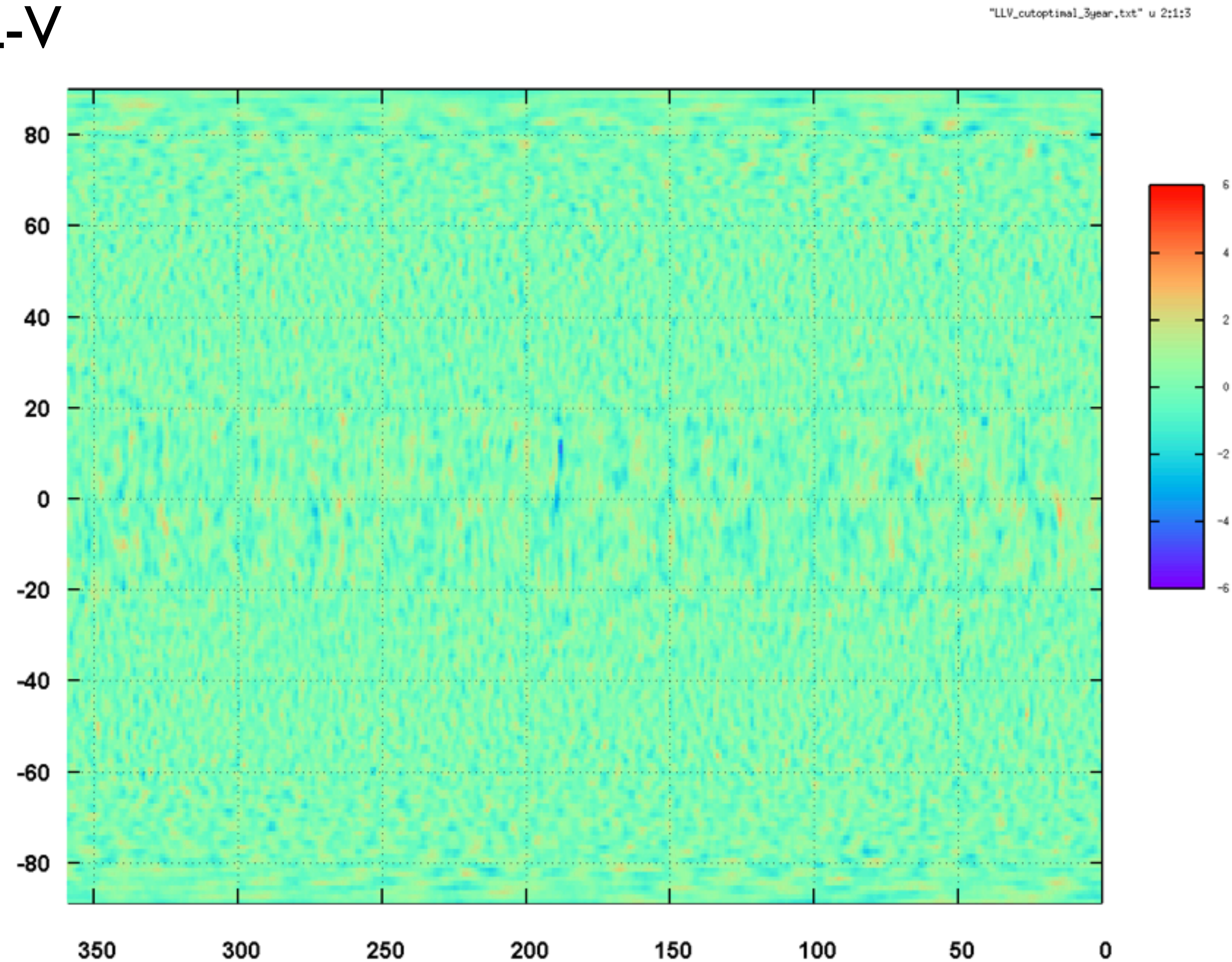
LL-V



"LLV_cutoptimal_3year.txt" u 2:1:3
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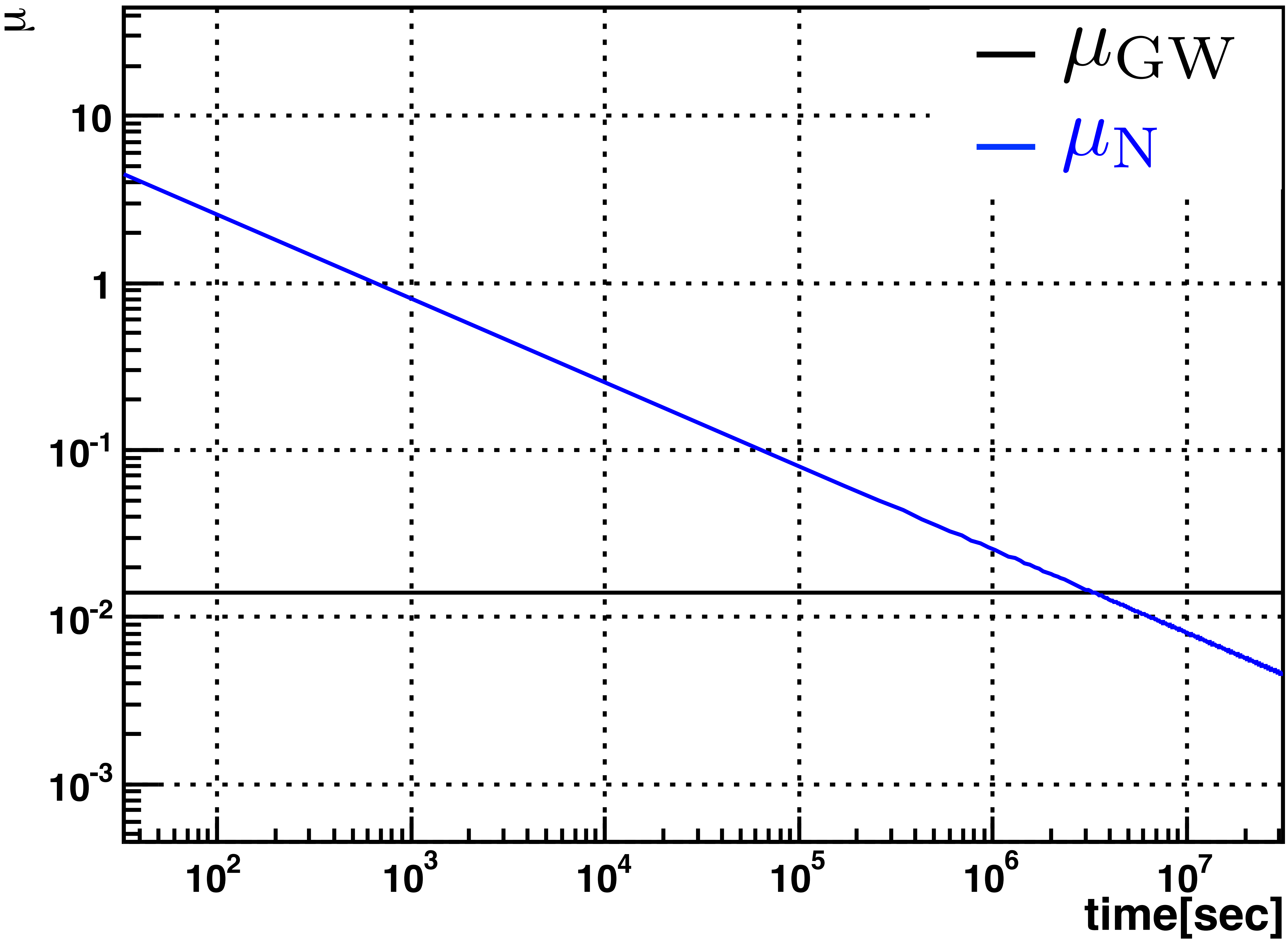
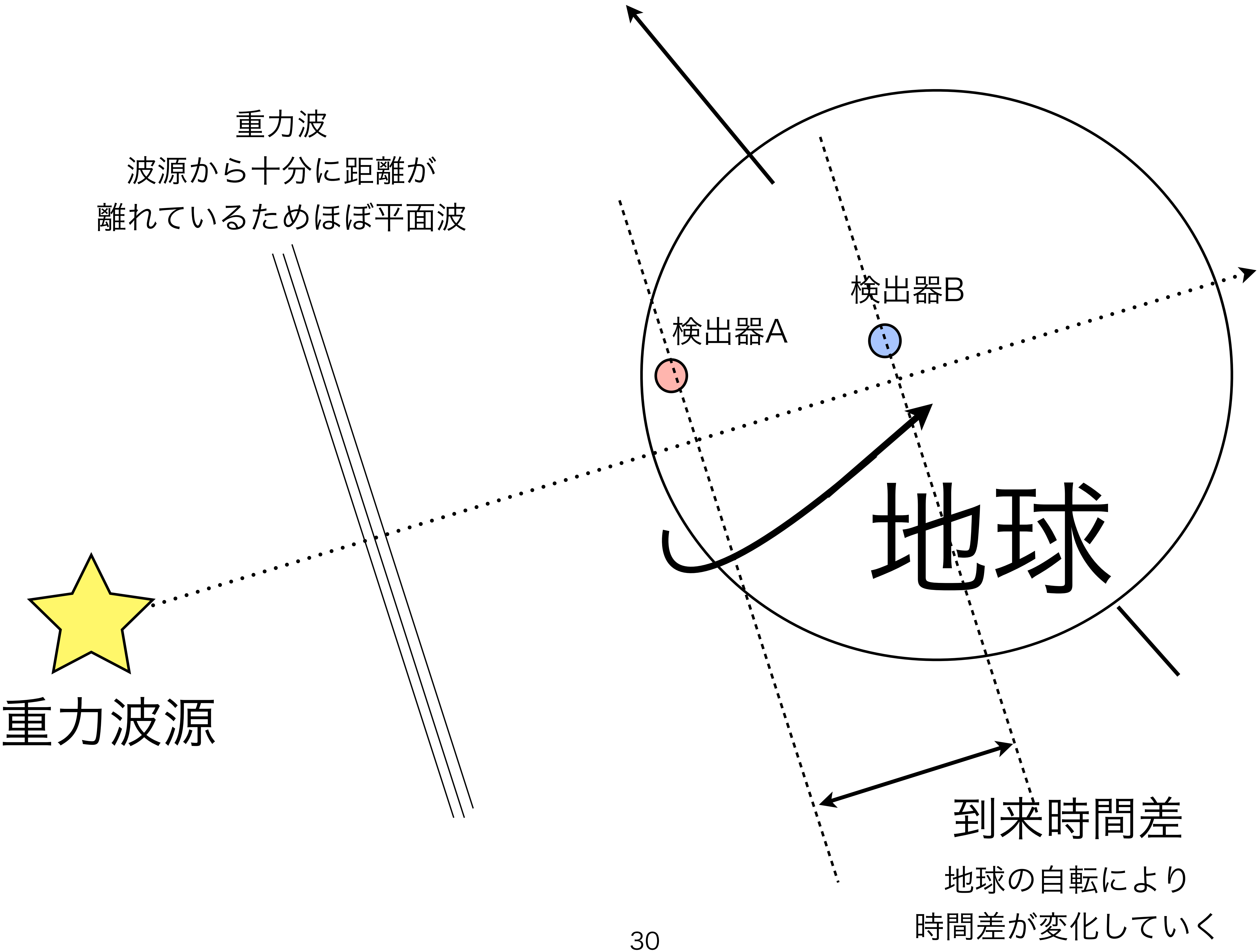
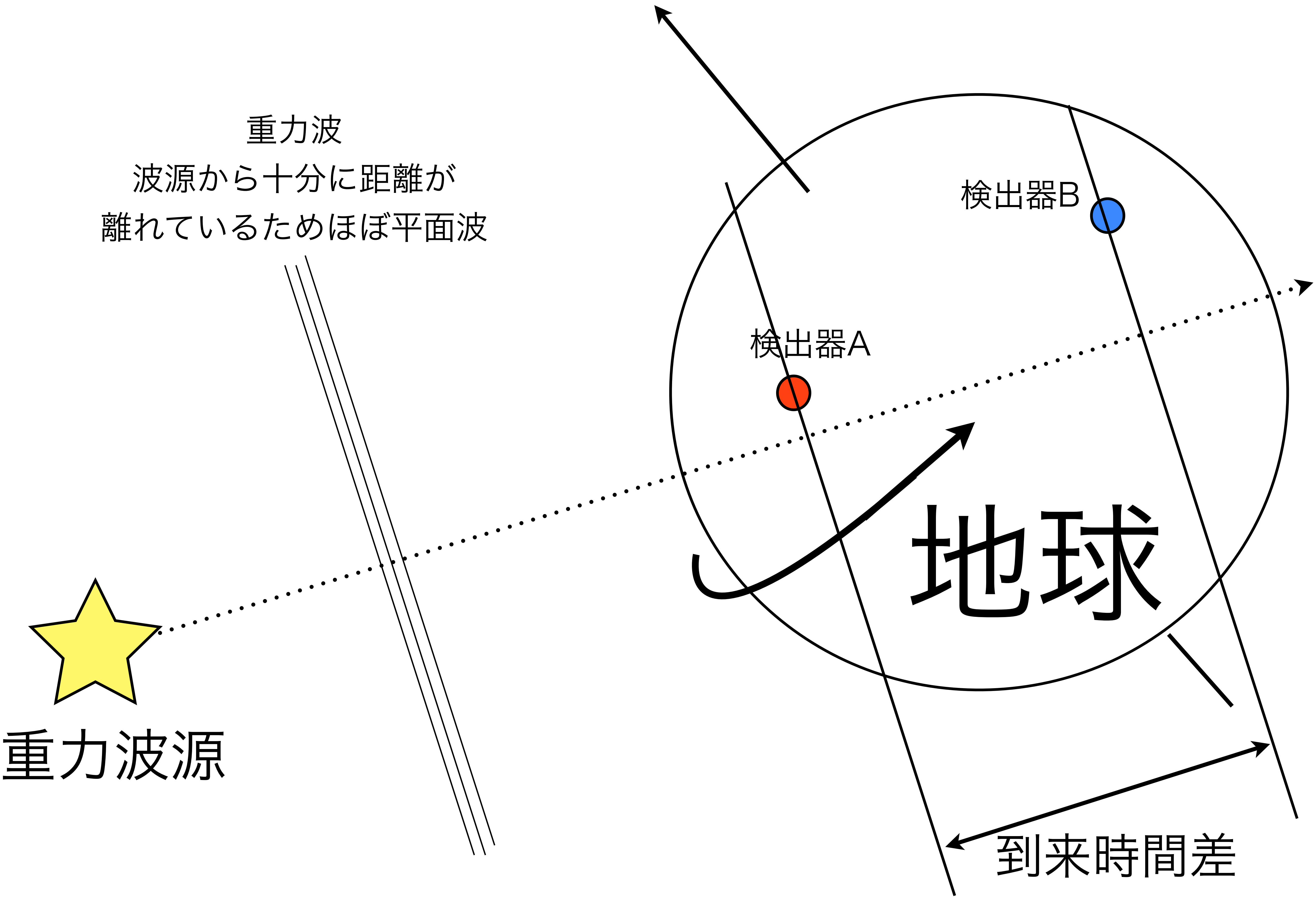


Y. Okada, master thesis(2012)


重力波
波源から十分に距離が
離れているためほぼ平面波
検出器B
検出器A
地球
重力波源
到来時間差
地球の自転により
時間差が変化していく


重力波
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