MCMC Parameter Estimation with Frequency-Domain Inspiral Waveforms

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- Results of MCMC parameter estimation
- Summary and Future work

Waveform models

TaylorF2 (F2): the only frequency-domain inspiral waveform used in the LAL detection pipeline

Standard F2 : 3.5pN phase + Newtonian amplitude

Latest progress: known physical effects are implemented.

We focus on amplitude corrections (2.5pN, TaylorF2Amp)

TaylorF2

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)},$$

TaylorF2Amp

$$\begin{aligned}
\tilde{h}(f) &= \frac{M\,\nu}{D_L} \sum_{n=0}^{5} \sum_{k=1}^{7} V_k^{2+n} \left(k\,\frac{dF}{dt}\right)^{-1/2} \\
&\times \left(\alpha_k^{(n)}\, e^{i\,(2\pi\, f\, t(F) - k\,\Psi(F) - \pi/4)} + \beta_k^{(n)}\, e^{i(2\pi\, f\, t(F) - (k\,\Psi(F) - \pi/2) - \pi/4)}\right), \\
&= \frac{M\,\nu}{D_L} \sum_{n=0}^{5} \sum_{k=1}^{7} V_k^{n-\frac{7}{2}} \sqrt{\frac{5\,\pi}{k\,48\,\nu}}\, M\,\left(1 + \mathcal{S}_2\, V_k^2 + \mathcal{S}_3\, V_k^3 + \mathcal{S}_4\, V_k^4 + \mathcal{S}_5\, V_k^5\right) \\
&\times (\alpha_k^{(n)} + e^{i\,\pi/2}\,\beta_k^{(n)}) e^{i\,(k\,\Psi_{\rm SPA}(f/k) - \pi/4)}, \\
&= \frac{M^2}{D_L} \sqrt{\frac{5\,\pi\,\nu}{48}} \sum_{n=0}^{5} \sum_{k=1}^{7} V_k^{n-\frac{7}{2}}\, \mathcal{C}_k^{(n)} e^{i\,(k\,\Psi_{\rm SPA}(f/k) - \pi/4)}.
\end{aligned}$$

n: pN order

k: harmonic order

Injections

* non-spinning compact binary inspirals (9 parameters)

* S/N~20 (by adjusting the distance for a given chirp mass)

* BH mass = 10 M_{sun}, NS mass = 1.4 M_{sun}

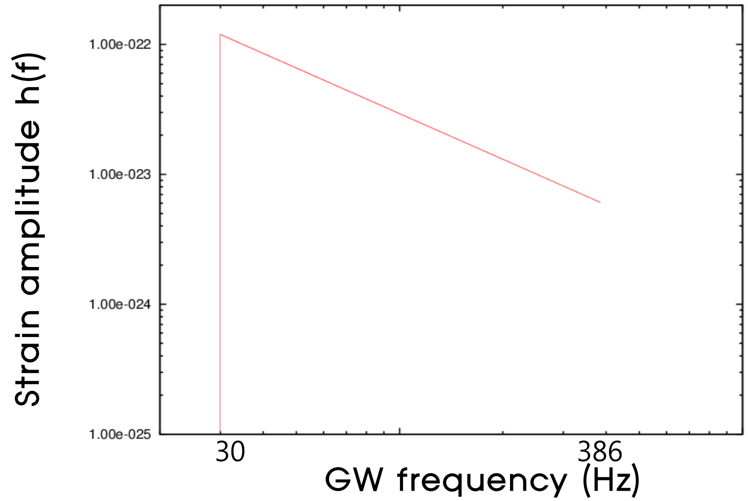
* Initial LIGO-Virgo (f_{low}=30Hz)

Binary	Chirp mass Mc (M_{sun})	Symmetric mass ratio η	distance (Mpc)	Inclination (rad)	Polarization (rad)	Orbital phase (rad)	Coalescence time (s)	RA (rad)	Dec (rad)
NS-NS	1.219	0.245	12.0	0.785	2.606	3.31	894383679	0.645	0.575
BH-NS	2.9943	0.1077	23.1						
BH-BH	8.705	0.245	45.0						

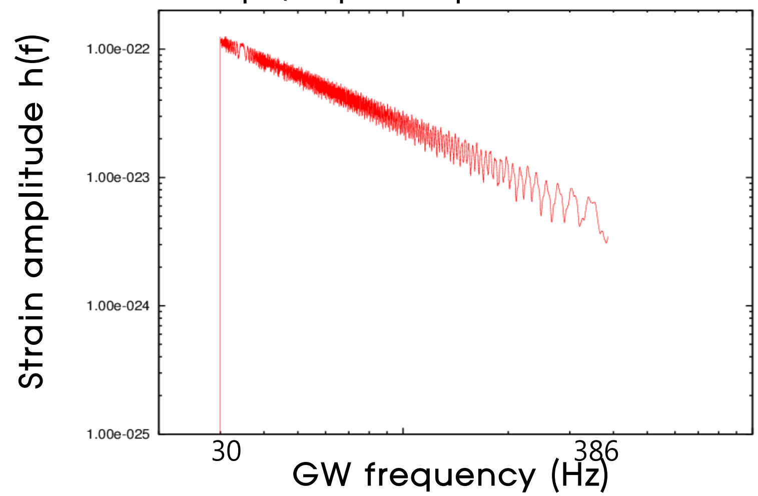
Effects of amplitude corrections

BH-NS inspiral (signal length ~ 12 s) mass ratio = 0.14

F2 (Newtonian)

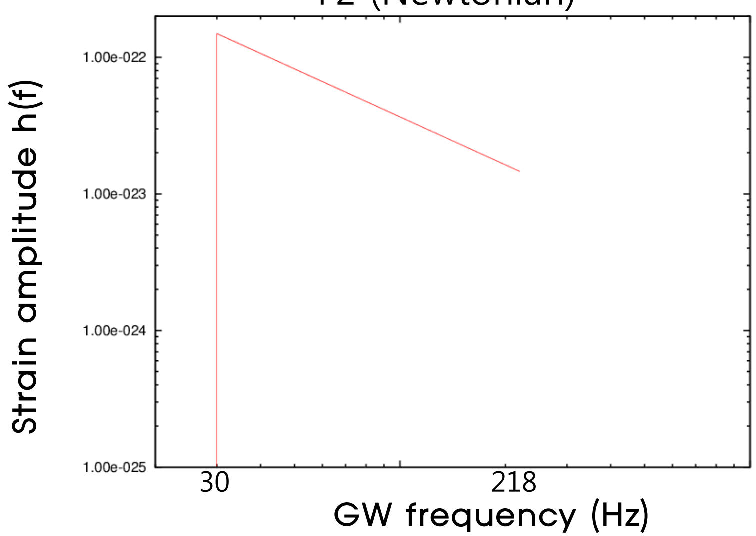


F2Amp (2.5pN amplitude corrections)

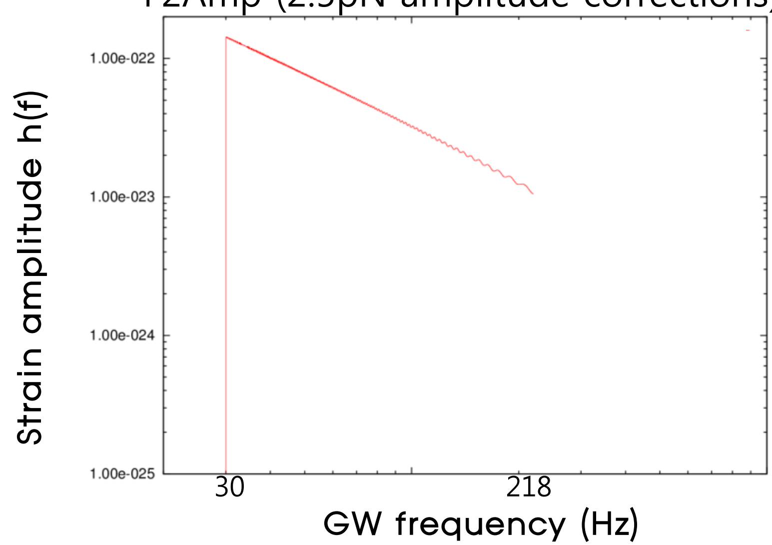


BH-BH inspiral (signal length ~ 2 s) mass ratio ~ 1.0

F2 (Newtonian)



F2Amp (2.5pN amplitude corrections)



MCMC computation time

- We measure the time to reach “convergence” of an MCMC chain
- For our injections, a few x $10^6 \sim 10^7$ iterations are required
- Typical computation time at KISTI cluster : days to months
- F2Amp template : 15 times longer CPU times than MCMC with F2

- Computer resource at KISTI GSDC
    - 35 nodes x 12 processors (Intel(R) Xeon(R) CPU X5450, 3.00GHz), REM: 2GB per cpu
    - OS: Scientific Linux 6 (LDG standard) , compiler: icc and gcc
    - scheduler: condor (LDG standard)
    - dedicated to KGWG research

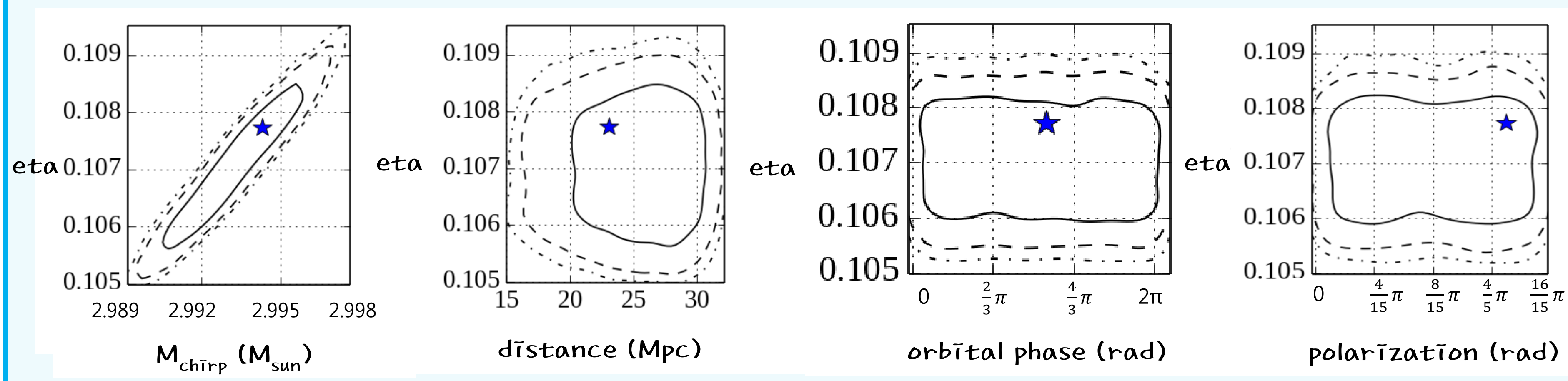
Results : 2-d contours for BH-NS inspirals

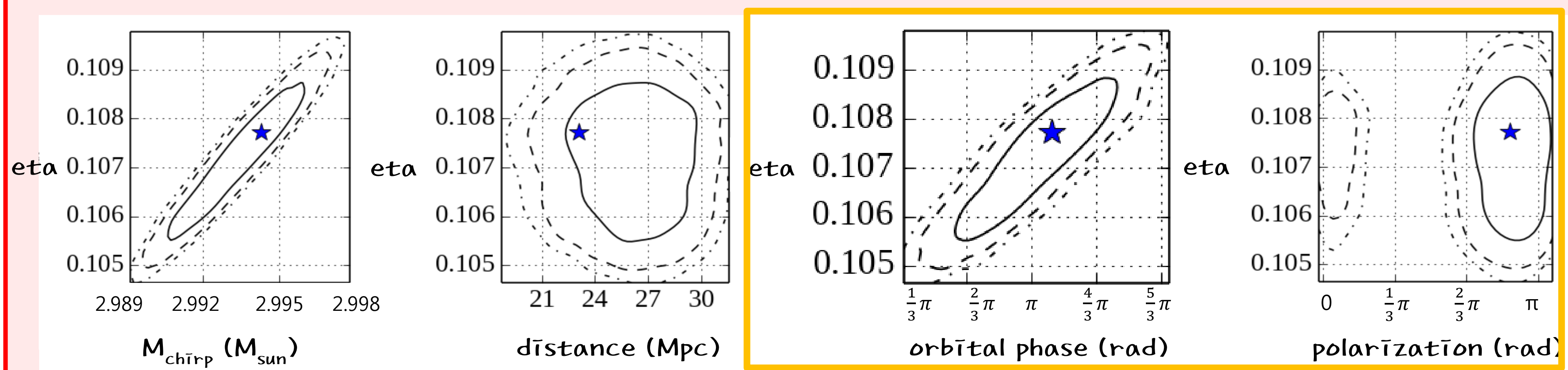
m1 = 10 M_{sun}
m2 = 1.4 M_{sun}
η ≈ 0.107
mass ratio ≈ 0.14

injection=template
If neglecting amplitude corrections,
orbital phase and polarization angles cannot be constrained.

★ : injection
— 67%
– – 90%
- · - · 95%

injection : F2 , template : F2 SNR = 23.8





injection : F2Amp, template : F2Amp SNR = 21.5

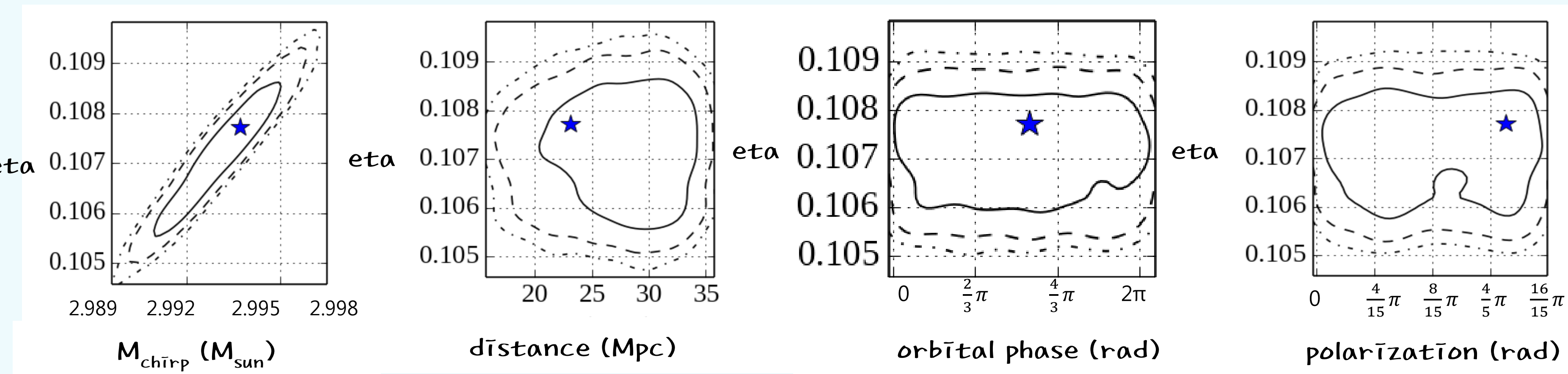
Results : 2-d contours for BH-NS inspirals

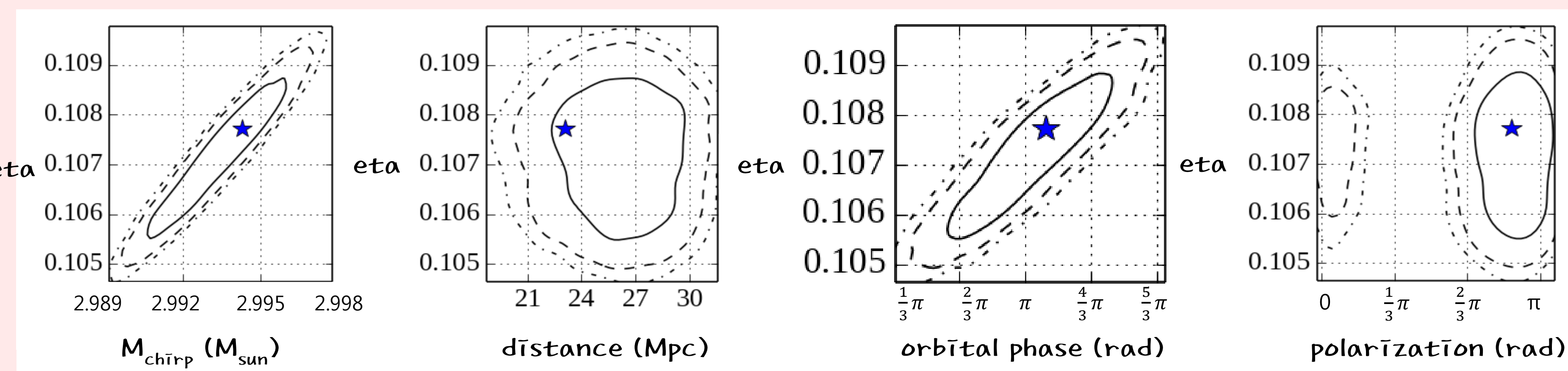
m1 = 10 M_{sun}
m2 = 1.4 M_{sun}
$\eta \approx 0.107$
mass ratio ≈ 0.14

injection=F2Amp, template=F2 or F2Amp
log (evidence) values are similar, but
orbital phase and polarization are constrained only with F2Amp templates.

★ : injection
— 67%
– – 90%
- · - 95%

injection : F2Amp , template : F2 log(evidence) = 196.5





injection : F2Amp, template : F2Amp log(evidence) = 197.1

Results : 2-d contours for BH-BH inspirals

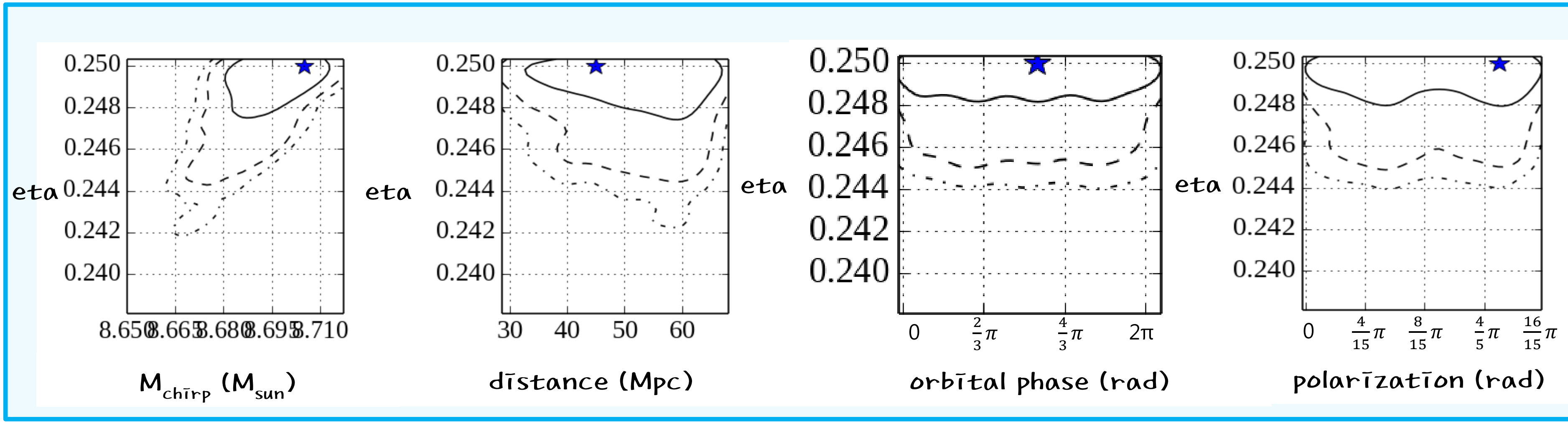
m1 = 10.1 M_{sun}
m2 = 9.9 M_{sun}
$\eta \approx 0.25$
mass ratio ≈ 1.0

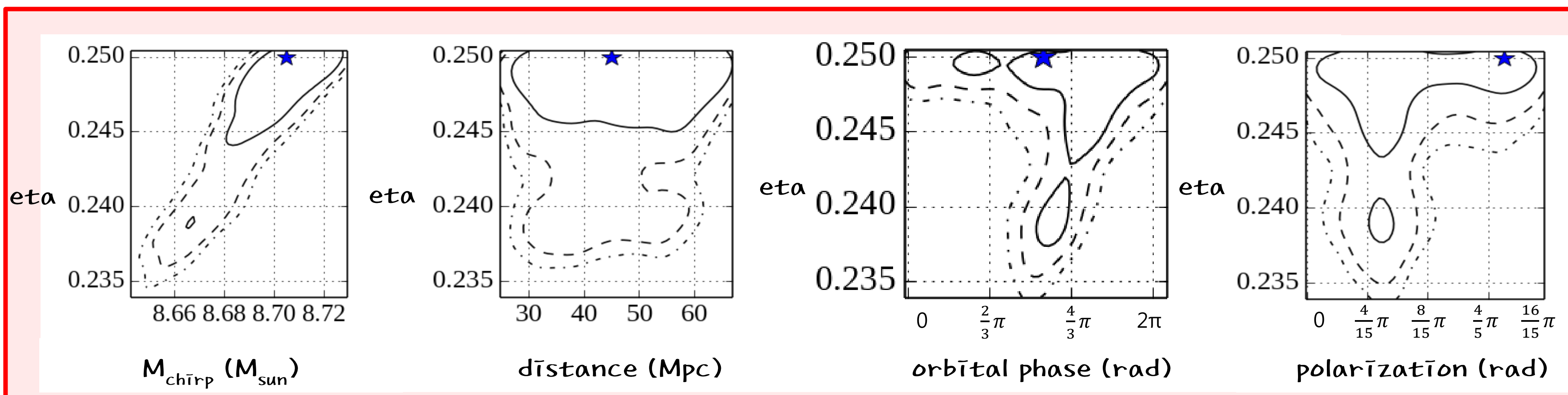
injection=template

amplitude corrections are more important for non-equal mass systems

★ : injection
— 67%
– – 90%
· – · 95%

injection : F2 , template : F2 SNR = 25.8



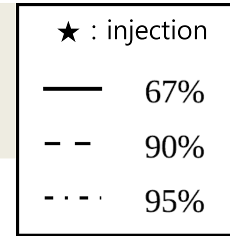


injection : F2Amp, template : F2Amp SNR = 23.1

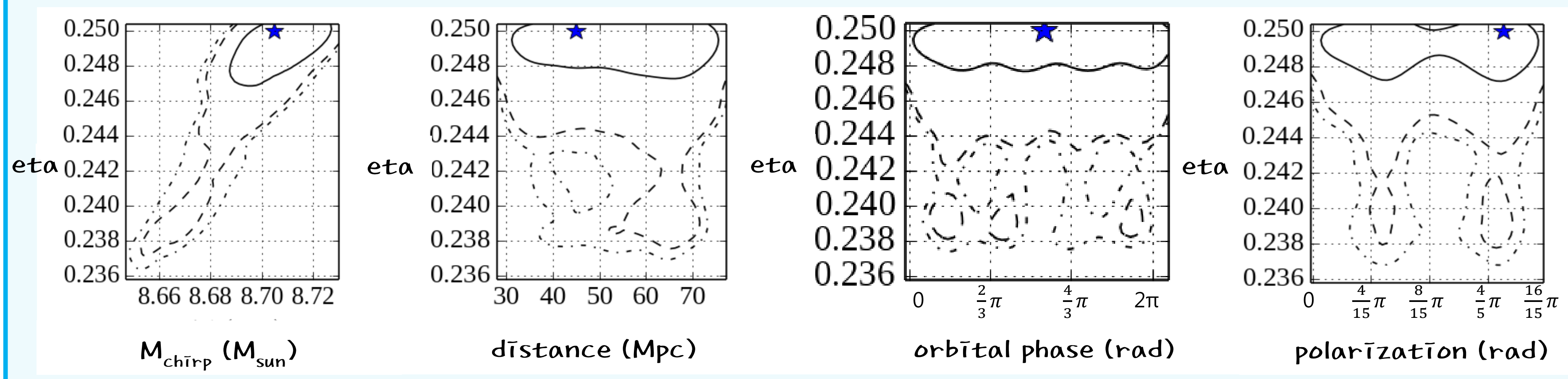
Results : 2-d contours for BH-BH inspirals

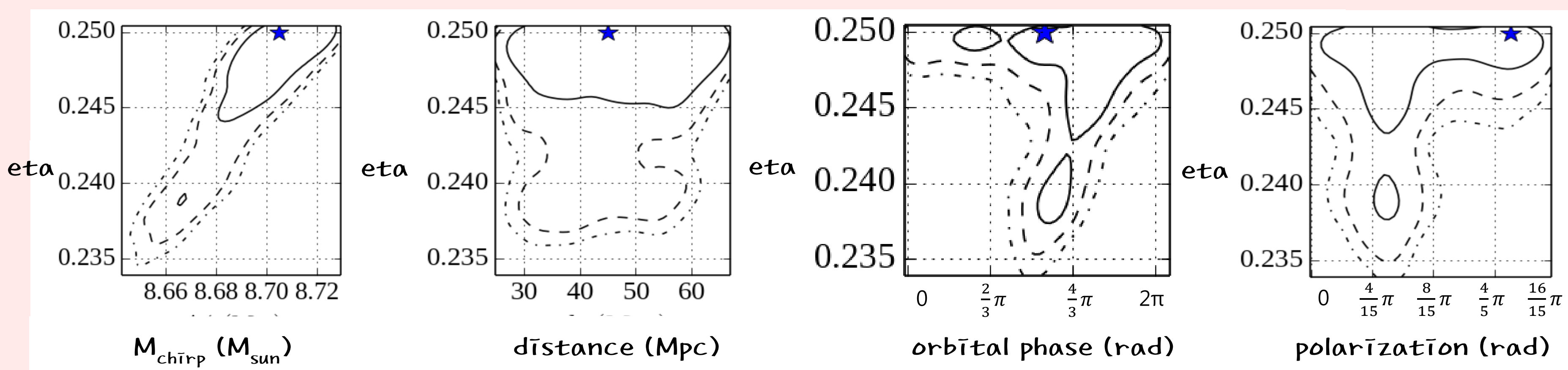
m1 = 10.1 M_{sun}
m2 = 9.9 M_{sun}
$\eta \approx 0.25$
mass ratio ≈ 1.0

injection=F2Amp, template=F2 or F2Amp



injection : F2Amp , template : F2   log(evidence) = 226.2

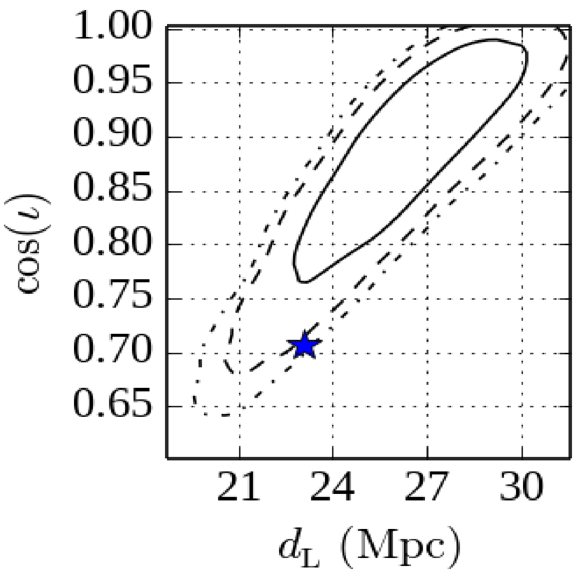


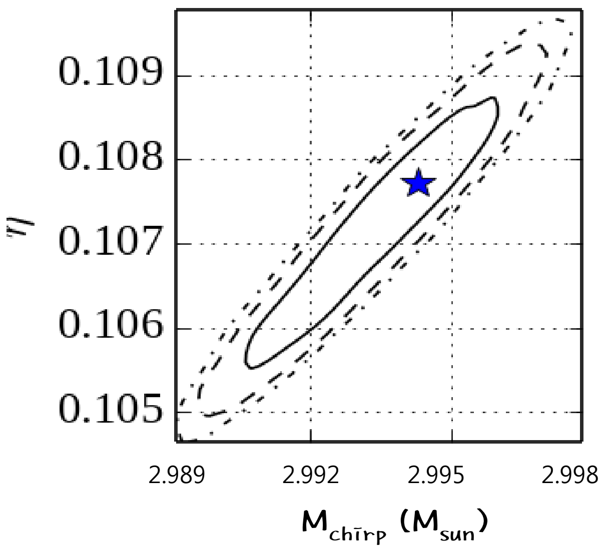


injection : F2Amp, template : F2Amp   log(evidence) = 236.9

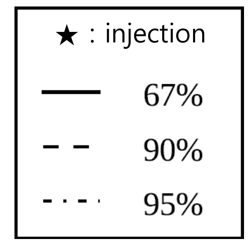
Comparison between F2Amp and SpinTaylorT4

non-spinning BH-NS inspiral with the initial LIGO-Virgo network
same injection parameters, same Gaussian noise realizations

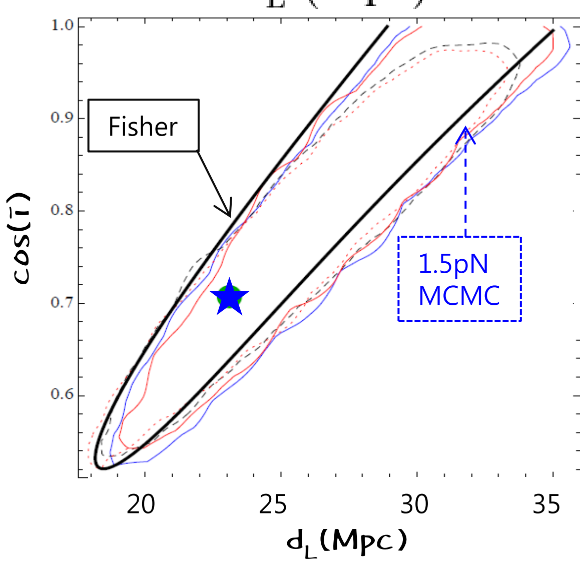


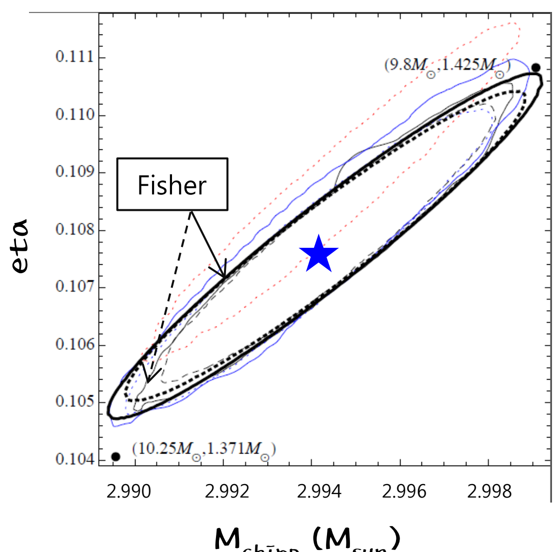


TaylorF2Amp
(2.5pN amplitude corrections)



MCMC only





SpinTaylorT4
(1.5pN amplitude corrections)

Cho et al. 2013, PRD, 87, 024004

All contours: 90% confidence interval
solid: Newtonian, dashed: 1.5pN

Fisher + MCMC

Summary

- We developed TaylorF2Amp and tested it with lalinference_mcmc (2.5pN amplitude + 3.5pN phase corrections)
- LAL parameter estimation library is partially reviewed with frequency-domain waveforms. Code profiling with lalinference_mcmc will be discussed in the afternoon (DAS technical session).
- MCMC results are similar to what was presented by Cho et al. (2013) with SpinTaylorT4 waveform: Amplitude corrections allow us to constrain orbital phase and polarization. Marginal improvements in mass, distance, inclination.
- Typical computation time: a few days (non-spinning, Newtonian, F2). When amplitude corrections (2.5pN) are turned on, computation time increases by a factor of 15!
- Only strong sources with different masses (if known from a detection pipeline) will be worth being analyzed with amplitude corrections.

Future Work

- To try more MCMC simulations for various injections
- To compare MCMC results and computation time
  - 1.5pN vs 2.5pN amplitude corrections with F2Amp
  - F2Amp vs time-domain waveforms
  - MCMC vs Fisher Matrix predictions with F2Amp
- To make aligned-spin F2 “injections” available for LALInference
- To keep working on the code optimization (of F2Amp), in order to reduce computation time with parameter estimation pipeline